Constructing Summary Indices of Quality of Life: A Model for the Effect of Heterogeneous Importance Weights
Michael R. Hagerty and Kenneth C. Land

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The authors consider how to construct summary indices (e.g., quality-of-life [QOL] indices) for a social unit that will be endorsed by a majority of its citizens. They assume that many social indicators are available to describe the social unit, but individuals disagree about the relative weights to be assigned to each social indicator. The summary index that maximizes agreement among citizens can then be derived, along with conditions under which an index will be endorsed by a majority in the social unit. The authors show that intuition greatly underestimates the extent of agreement among individuals, and it is often possible to construct a QOL index that most citizens agree with (at least in direction). In particular, they show that the equal-weighting strategy is privileged in that it minimizes disagreement among all possible individuals’ weights. They demonstrate these propositions by calculating real QOL indices for two surveys of citizens’ actual importance weights.

Keywords: summary index construction; quality-of-life indices; heterogeneous importance weights; well-being accounts

Sociologists have constructed summary indices for the comparison of social units (e.g., cities, states, nations) with respect to multiple dimensions of social life at least since the mid-twentieth-century work of Angell (1942, 1947, 1949, 1951, 1972) on the social and moral integration of American cities. The past decade has seen increased interest among

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sociologists and other social scientists in the construction of summary or composite indices of social well-being or, as they have come to be termed, quality-of-life (QOL) indices. This work coincides with a general interest in the subject among individuals, policy makers, and political leaders. For instance, the term quality of life has been invoked on the floor of the U.S. Congress more than 20 times per week in recent years (Government Printing Office 1999), and the National Academies are working with Congress to develop key national indicators (Government Accounting Office 2003).

There is, however, little agreement among sociologists and other social scientists on methods for aggregating social indicators to create a QOL index that is useful for public discourse on social well-being and policy issues relevant thereto. Some researchers even argue that no summary index should ever be computed (Johansson 2002; Erickson 1993). They cite two important barriers to QOL indices. The first is that the concept of QOL is too general to be useful. Critics point to the problem that QOL is a composite indicator whose components (e.g., crime rate, gross domestic product [GDP]/capita, environmental damage) are not highly correlated, nor are their causes identical. Hence, traditional factor analysis would recommend that these components be treated as separate factors. While these diverse components probably should not be combined into a single first-order factor, it is possible that QOL could be considered a higher-order factor (a factor analysis of first-order factors). Moreover, a QOL index can be useful in considering how people make emigration decisions (“Is the QOL of one state higher than my current residence?”) and in how people make political decisions (“Am I better off today than 4 years ago?”). These decisions require individuals to integrate many objective indicators into a single subjective evaluation of whether they should move or whether they should agitate against the incumbent. QOL research has shown (Veenhoven 1994; Diener and Seligman 2004) that people in many nations are able to form reliable judgments of their subjective QOL (often measured by national surveys as overall satisfaction or happiness with life as a whole) as a function of objective indicators such as personal income, political freedom, degree of physical health, marital status, race, inequality, and other variables. Despite the cognitive difficulties in combining these various objective indicators, people appear to do so reliably and to use their judgments in important social decisions.

Finally, a QOL index can be very useful to policy makers, who need to know the relative contribution of each social indicator (e.g., unemployment, crime rate, divorce) on citizens’ perceived well-being, so that they can target interventions and budgeting to each indicator area. For example,
economists have recently shown that the national unemployment rate is more important to overall QOL than the national inflation rate (Di Tella, MacCulloch, and Oswald 2001). Therefore, this article addresses issues raised by the stream of research (Inkeles 1993; Land 2000; Hagerty et al. 2001) that considers QOL to be a measurable and useful concept.2

A second fundamental reason for questioning the usefulness of QOL is that individuals, policy makers, and researchers themselves disagree on the relative importance weights3 that each social indicator should have in a summary QOL index. Without agreement on the importance of each social indicator, chances for agreement on the overall QOL index would seem slim. Social science research can reduce disagreement substantially by establishing the effects of various objective variables on citizens’ QOL (measured by questions on life satisfaction or happiness; see Veenhoven 1994 and Diener and Seligman 2004). But such research can never completely eliminate heterogeneity in subjective weights because each individual deviates somewhat from the average effect (this is the rationale underlying random-effects models). For example, people will always differ on how much they value additional income due to individual differences, such as how materialistic their values are (Diener and Seligman 2004). Hence, the common call for “doing more research” is not likely to eliminate all heterogeneity in individuals’ judgments of QOL. Instead, it is useful to pose the following question: How much heterogeneity is possible in a society wherein a majority of members still can agree on a single QOL index? Some minimal level of agreement is necessary in every society to pursue shared goals.

A formal analysis has not previously been done on how heterogeneity affects chances for agreeing on a social index. This article fills this gap and proves results that are not predicted by intuition. In particular, the present article (a) specifies a model for how individuals disagree with each other on QOL judgments, (b) predicts how much disagreement results from various types of QOL indices and various distributions of weights, and (c) recommends QOL weights that maximize agreement among individuals.

We hasten to note that much research on social well-being can be conducted without any overall composite or summary index of QOL—by examining individual components of quality of life (e.g., public health, education, income, etc.). It is more parsimonious to avoid assuming any higher-level construct when interest is restricted to one component of QOL or when all lower-level components agree. For excellent examples of such research, see Inkeles (1993) for the effect of modernization on QOL, Weede (1993) for the effect of democracy on QOL, and Stokes and
Anderson (1990) for the effect of disarticulation on QOL. But when lower-level components of QOL disagree in sign, the inevitable question arises: What is the net effect of these conflicting social indicators on individuals’ QOL? This query sometimes is posed more brutally by individuals and politicians with the question “Are we better off than x years ago?” To answer this type of question, people must transform the many objective indicators (such as unemployment, political freedom, and crime rate) into a subjective judgment of overall QOL. Psychologists term this process the psychophysical transformation. The current article models this transformation using previous work in QOL reviewed by Veenhoven (1994) and Diener and Seligman (2004) and examines implications for societal agreement. If some level of societal agreement exists, then it will be easier to create political agreement on which QOL indicators to target for budgeting and intervention (Ferriss 1988). Moreover, publicly available QOL indices could provide powerful shorthand descriptions for overall trends in QOL, much as the Dow-Jones Industrial Average is a powerful public index of the performance of more than 5,000 stocks in the United States.

**Previous Research on QOL Indices**

Land (2000) documents the rapid growth of QOL indices in his review of the field of social indicators for the *Encyclopedia of Sociology*:

With the tremendous increase in the richness of social data available for many societies today as compared to two or three decades ago, a new generation of social indicators researchers has returned to the task of summary index construction. Some examples: (1) at the level of the broadest possible comparisons of nations with respect to the overall quality of life, the *Human Development Index* (United Nations Development Programme 1993), *Diener’s (1995) Value-Based Index of National Quality-of-Life, and Estes’ (1988; 1998) Index of Social Progress*; and (2) at the level of comparisons at the national level over time in the United States, the *American Demographics Index of Well-Being* (Kacapyr 1996), the *Fordham Index of Social Health* (Miringoff 1996), and the *Genuine Progress Indicator* (Redefining Progress 1995). (p. 2687)

The QOL indices he cites vary on a number of indicators: whether they incorporate only “objective” indicators such as crime rate or “subjective indicators” such as social surveys, whether they are cross-sectional (multiple countries at one point in time) or time series (one country at multiple
points in time), and the weights they assign to social indicators. The current article uses two examples that Land (2000) cites to show how weights can be developed for social indices. Each will be briefly described here and then further analyzed later.

**Human Development Index.** The Human Development Index (HDI) is an example of a QOL index that can be used to make cross-sectional comparisons among social units—in this case, nations. (Recent versions also report time-series trends over 25 years, so it can now be used for time-series analyses as well.) The HDI is published annually by the United Nations Human Development Program. It is calculated from three social indicators: log (GDP) in purchasing power parity, life expectancy in years, and education (a weighted average of literacy rate and school enrollment rate). These three indicators are first transformed so that their ranges are equal and then are averaged (with equal weights) to derive the HDI index. An HDI score is calculated for each nation for which data on these three indicators are available. Nations then are arrayed from the most to the least developed with respect to these indicators of human development. The annual United Nations Human Development Program reports do not justify why the indicators are weighted equally. How much would the HDI change if the weights change? Do individuals (or members of any social group) hold equal weights for those indicators? Do individuals hold such diverse weights that no index can capture the views of the group? Unless we know the answer to this, computing a summary index seems premature.

**Index of Social Health.** The Index of Social Health (ISH) was developed by Miringoff and Miringoff (1999). Using the United States as the social unit to be indexed, the ISH is based on 16 social indicators tracked annually from 1970 to the most recent year available: average weekly earnings, life expectancy at age 65, gap between rich and poor, violent crime rate, infant mortality, child abuse, children in poverty, teenage suicide, drug abuse, high school dropout rate, teenage births, unemployment, health insurance coverage, poverty among those older than 65, alcohol-related traffic fatalities, and housing affordability. Rates for these indicators for any specific year are indexed as percentages of their values for the year in which they had their “best practice” or best performance value. They are then averaged with equal weights to obtain the value of the ISH for a specific year.

Hagerty et al. (2001) review not only the HDI and ISH but also 20 additional QOL indices, and they conclude that none satisfactorily addresses
the problem of weighting because none explicitly considers how individuals themselves weight each social indicator, nor do they deal with the likely variation in weights over people. The most common weighting procedure is to use “equal weights” for all social indicators (after each has been standardized). Equal weighting is used not only by the HDI and the ISH but also by Diener’s (1995) Value Based Index of National Quality of Life; Estes’s (1984, 1988, 1997) Index of Social Progress; Johnston’s (1988) Comprehensive Quality of Life Index; Land, Lamb, and Mustillo’s (2001) Index of Child and Youth Well-Being; Morris’s (1979) Physical Quality of Life Index (PQOL); and Veenhoven’s (1996) Happy Life-Expectancy Index. Many of these indices apply equal weights without stating why, and none consider whether individuals themselves would weight these components equally.

The next most common weighting procedure is to use factor analysis to weight components. Money magazine’s “Best Places to Live” uses a combination of factor analysis and surveys of readers’ importance weights for 40 components of QOL. However, Guterbock (1997) shows that the economic factors are greatly overvalued in their index because their survey includes more items related to economics, despite the fact that readers rated crime, environment, and health as more important than economics. Another index that uses factor analysis is Estes’s (1988) Index of Social Progress, which uses a two-stage varimax factor analysis to assign weights to 40 indicators. The basic difficulty with using factor analysis is that the weights are derived to maximize the variance explained in the social indicators, without any reference to individuals’ weights. If items are carefully sampled from individuals’ and decision makers’ concerns, then this practice can be a proxy for weights. But no QOL index specifically adopts this practice. Guterbock (1997) concludes, “The relative weights given to economics and the other eight factors should be made part of the research problem. They should not be decided in advance by editorial fiat or as an inadvertent by-product of initial questionnaire design” (p. 355). In this article, we provide a framework to jointly consider weights and social indicators as part of the research problem of constructing a QOL index that will be approved both by individuals and by researchers.

The problem of weighting becomes even more pressing when comparing alternative indices because alternative indices often use quite different social indicators and show different trends for countries. For example, Miringoff and Miringoff’s (1999) ISH uses 16 indicators, none of which overlaps with the HDI indicators. (The ISH does not include any indicator of GDP/capita and uses different indicators for education
and life expectancy.) Although the HDI index concludes that QOL in the United States is increasing, the indicators from the ISH show that QOL in the United States is decreasing. Clearly, the choice of weights for social indicators is crucial to its acceptance by individuals and policy makers.

We should point out that agreement with individuals’ judgment is not the only criterion for a good QOL index. There are good arguments for considering some indicators that individuals do not consider, if sociologists identify them as leading indicators of QOL (social capital, deterioration of child-rearing practices). Other criteria for QOL indices are listed in Hagerty et al. (2001) and include reliability, concurrent validity, and so on. But QOL ultimately must be assessed by individuals, to whom the United Nations Charter guarantees self-determination.

In this article, we examine how much indices vary in the presence of a distribution of different people’s values. In the next section, we specify a model for how individuals disagree with each other on QOL judgments and develop a formula for computing the correlation between any two weighting systems. These results then are applied to existing QOL indices for the first time to assess agreement among them. We also examine empirical distributions of individuals’ values in 48 different nations. A final section concludes with recommendations for constructing QOL indices.

**A Model of Agreement Between Two QOL Indices When Weights Differ Among Individuals**

Define $X$ as a matrix with $K$ columns and $N$ rows. The columns record the scores from $K$ social indicators (e.g., GDP/capita, Gini coefficient of income inequality, divorce rate, etc.) on each of the $N$ social units (e.g., cities, states, nations). Define $W_i$ as the weighting (column) vector of individual $i$, measuring how important each social indicator is to that person. Then $i$’s QOL judgment of social unit $n$ is the sum of the $K$ social indicators, weighted by person $i$’s importance weights for each indicator $k$ or, for short, *importances*:

$$Q_{in} = \sum w_{ik} x_{kn}, \quad w_{ik} > 0, \quad \text{for } n = 1, \ldots, N \text{ social units}$$

(1)

Although this model may appear restricted to linearity, it can also incorporate nonlinear effects by adding a new variable that is some function of the old indicator (e.g., log [GDP/capita], as in the HDI). The general additive model has been successful at approximating many more
complex functions, and Sastre (1999) reports that people use an additive model in direct tests of how people judge others’ quality of life.

We constrain each weight to be a nonnegative number (i.e., only positive or zero weights are allowed). Hence, we assume that any indicators that are negatively related to QOL (e.g., infant mortality) are reversed in sign to allow positive importance weights. This assumption that everyone has positive weights is probably not controversial for social indicators such as GDP/capita and infant mortality, where everyone prefers more money and better health, given that all else is held constant. But it may be controversial for indicators such as divorce rate, where some people may view a higher divorce rate as reflecting more freedom for women, but others view it as a decline in support for children. In such a situation, one could add indicators for the omitted variables (women’s freedom, support for children) to ensure that weights are positive for all individuals.

Note that multiplying all weights by a constant $c$ simply expands the QOL index by the factor $c$ and does not change the ordering of the social units being rated. Therefore, without loss of generality, we divide each person $i$’s weights by $\sum w_{ik}$ so that all $k$ weights sum to 1 for each $i$.

Finally, the linear model in equation (1) should not be confused with the simple utilitarian model of Bentham, where utilities of individuals are summed to get social welfare, ignoring inequality among individuals. In contrast, equation (1) allows some of the indicators to be measures of the overall stratification of income or wealth (positional information), as well as measures of individuals’ freedom (called nonutility information by Sen 1993). Hence, the model in equation (1) offers great flexibility in modeling individuals’ evaluations of QOL.

As noted previously, QOL indices may be used either in cross-sectional or over-time comparisons. The goal in cross-sectional comparisons is to evaluate a social unit relative to other social units. This may, for example, allow people to decide in which nation they should live (e.g., the International Living Index) or which nation is in more need of development assistance (e.g., Estes’s Index of Social Progress). In the case of over-time comparisons, QOL indices rate multiple time periods in the same social unit (time series) to decide whether QOL has increased or decreased over time in that entity. The goals in the time-series case are to provide information for informing individuals about QOL changes over time, to fuel a public policy debate, and to decide whether policies are successfully improving QOL within a given country (though, of course, many uncontrolled variables will also influence QOL). It is desirable to find a measure of agreement that will be useful in both of these cases.
For this purpose, we propose to use the familiar Pearson correlation coefficient to measure levels of agreement between the QOL judgments of individuals \( i \) and \( j \), denoted by \( A_{ij} \). The correlation coefficient has a number of desirable properties for measuring agreement. It has finite limits between \(-1\) (complete disagreement) and \(1\) (complete agreement), and its statistical properties are well researched. It already has been widely used as a measure of interrater agreement and as a measure of similarity between persons in cluster analyses. Another attractive property of the Pearson correlation coefficient is that certain values represent important levels of agreement among people. The first is naturally \( A_{ij} = 1 \), where perfect agreement occurs between the QOL indices of \( i \) and \( j \). The second is \( A_{ij} = .7 \), which is the common cutoff among researchers for assessing agreement between raters. Agreement between raters is not expected to be perfect, but the .7 cutoff implies that about 50 percent of the variance in one rater should be predictable from the other (\( r^2 > .5 \)). The third noteworthy level of \( A_{ij} \) is zero, because this is the point above which the QOL index of individual \( i \) agrees in direction with that of \( j \). To see this, take the limiting example of QOL evaluations of year \( t \) and year \( t + 1 \). Then, \( A_{ij} > 0 \) implies that the raters agree on whether the nation’s QOL has increased or decreased during that time. This is a fundamental question that often helps define similar political parties, social classes, and interest groups. In the next section, we use the Pearson correlation coefficient to calculate the agreement between any two individuals whose importance weights differ among social indicators.

**Agreement Between QOL Indices When Importance Weights Differ**

The weighted sum in equation (1) is more compactly designated in matrix notation as

\[
Q_i = XW_i,
\]

where \( Q_i \) is an \( N \times 1 \) column vector of summary (or composite) index values (or scores) of individual \( i \) for each of the \( n = 1, \ldots, N \) social units; \( X \) is an \( N \times K \) matrix of values of the \( K \) social indicators for each of the \( N \) social units; and \( W_i \) is a \( K \times 1 \) column vector of weights of person \( i \) for the \( K \) social indicators in \( X \). We assume, without loss of generality, that each social indicator in \( X \) has already been standardized, so that the mean of each column of \( X \) is zero, and standard deviation is 1. The resulting composite scores \( Q_i \) will also have a zero mean since the original indicators had zero
But in general, the composite scores will not have a standard deviation of 1. Our goal, then, is to find the correlation $A_{ij}$ between the QOL indices of individuals $i$ and $j$, with different weight vectors, $W_i$ and $W_j$.

By definition of the correlation coefficient,

$$A_{ij} = 1/(N - 1)S_i Q_i^T Q_j S_j,$$

where $N$ is the number of social units rated, $S_i$ is the inverse of the standard deviation of the QOL index for person $i$ (used to standardize the scores $Q_i$), and $Q_i^T$ denotes the matrix transpose of $Q_i$. The term $1/(N - 1)Q_i^T Q_j$ is the covariance of the QOL indices, which, after standardization, is the correlation coefficient between $Q_i$ and $Q_j$.

### Sample Calculations With the Model

Table 1 demonstrates some sample calculations with the model for two social indicators, each observed at three time periods, and two citizens $i$ and $j$. We first demonstrate the “worst possible case” for agreement, where the citizens have directly opposing weights, and the social indicators

<table>
<thead>
<tr>
<th>$X_i$</th>
<th>$X_2$</th>
<th>$W_i$</th>
<th>$W_j$</th>
<th>$Q_i = XW_i$</th>
<th>$Q_j = XW_j$</th>
<th>$A_{ij}$</th>
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<tr>
<td>-1</td>
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Note: (a) shows the “worst-possible case,” where weights are opposing and agreement is minimum at $-1$. (b) shows less extreme example, where weights are a mixture of $X_1$ and $X_2$, and the two indicators are correlated near zero, with resulting agreement of $+.67$. 
are negatively correlated. The first two columns of Table 1a show the observations of social indicators over the three time periods, where $X_1$ is rising over time and $X_2$ is declining. Note that both are reported in standardized scores. The next two columns of Table 1 show the weights for $i$ and $j$, where $i$ places all importance on the first indicator, and $j$ places all importance on the second. The next two columns calculate the resulting quality-of-life $Q$ judged by $i$ and $j$, and the last column shows their correlation $A_{ij}$. Agreement is at the minimum of $-1$ in this case because person $i$ perceives his or her QOL to be consistently increasing over time, but person $j$ perceives his or her QOL to be consistently decreasing over time.

While Table 1a demonstrates the worst possible case for agreement, Table 1b shows a less extreme example. Here, the social indicators are not correlated at $-1$ but at approximately $0$, and person $i$ gives the most weight to the first social indicator, while person $j$ gives the most weight to the second. Surprisingly, agreement here is calculated as $+0.63$, far from the $-1$ of the previous example. How much of the increase in agreement is due to the change in weights, and how much is due to the change in correlation between social indicators? How would agreement change in even more realistic conditions where entire distributions of citizens are considered and larger numbers of social indicators are added? In the next section, we derive proofs for these questions and show that the “worst case” in Table 1a is rare indeed.

**Analysis of the Model**

We now state several propositions that summarize properties of this model for measuring agreement between QOL indices for different individuals. When the propositions can be derived discursively and with no advanced mathematics, they are done so as part of this text. For Propositions 3 and 5, the mathematical arguments necessary to prove the propositions are more demanding and thus appear in the appendix.

Equation (3) can be simplified by substituting definitions of $Q_i$ from equation (2):

$$A_{ij} = 1/(N - 1)S_i(XX_i)^TW_jS_j$$

$$= 1/(N - 1)S_iW_i^T(XX^T)W_jS_j = S_iW_i^TR_xW_jS_j,$$

where the prime denotes matrix transposition, and $R_x$ is the $K \times K$ matrix of correlations among the $K$ social indicators. Next, we can expand on the
definition of the inverse of the standard deviation for the $S_i$ in equation (4) by using equation (2):

$$S_i = [(1/(N-1) \Sigma_{in} Q_{in}^2)]^{-1/2} = (1/(N-1)(XW_i)^T XW_i)^{-1/2} = (W_i^T R_x W_i)^{-1/2}. \quad (5)$$

Hence, equation (3) can be rewritten as

$$A_{ij} = W_i^* R_x W_j^*, \quad (6)$$

where

$$W_i^* = W_i / (W_i^T R_x W_i)^{1/2} \quad (7)$$

denotes the standardized weights vector. This shows that the correlation among summary indices $i$ and $j$ can be written as a function of the matrix of correlations among the original social indicators $R_x$ and some normalized function $W_i^*$ of the weights. Equation (6) shows that $W_i^*$ is proportional to the original weights $W_i$ but is adjusted by the squared weights and covariances to yield a standard deviation of 1 on the new QOL indices.\(^{10}\) This gives rise to the following:

**Proposition 1:** The correlation $A_{ij}$ between any two individuals’ QOL indices is a function not only of the two individuals’ weights but also is moderated by the correlations among the social indicators $R_x$.

In fact, we will show that, even when two persons’ weights are diametrically opposed, $A_{ij}$ can be surprisingly high because $R_x$ acts as a lower limit on agreement. We begin by examining the common situation where all correlations in $R_x$ are positive. For the simplest two-indicator case, the matrix notation in equation (6) can be expanded to

$$A_{ij} = w_{i1}^* (w_{j1}^* + w_{j2}^* r) + w_{i2}^* (w_{j1}^* r + w_{j2}^*). \quad (8)$$

All standardized weights $w^*$ are nonnegative because the raw weights themselves are always nonnegative.\(^{11}\) If the correlation $r$ in equation (8) is also nonnegative, then all variables in equation (8) are greater than (or equal to) zero, requiring that equation (8) be greater than (or equal to) zero. More generally, for any number of social indicators, the matrix multiplication in equation (6) can always be expanded as sums and products of $w_{ik}^*$ (always nonnegative) and $r_{ij}$. This leads to the important result:
**Proposition 2:** When all correlations among the $K$ social indicators $R_x$ are positive, then all individuals will agree on the direction of the QOL index $(A_{ij} > 0)$, regardless of the distribution of weights.

This result will be useful in later examples because many QOL indices have social indicators that are all positively correlated.

To generalize further, some elements in $R_x$ may be negative, so that $A_{ij}$ may be less than zero. How low (and how high) can agreement go, and under what conditions is agreement lowest (highest)? To answer these questions, we calculate both the minimum and the maximum $A_{ij}$:

**Proposition 3:** The maximum agreement $A_{ij}$ is 1 and occurs when $W_i = W_j$ (when the individuals’ weights agree). The minimum value of $A_{ij}$ for two social indicators ($K = 2$) is $r$ and occurs when $W_i$ is orthogonal to $W_j$ (i.e., $W_i = [1, 0]^T$ and $W_j = [0, 1]^T$ so that each individual places all his or her weight on different indicators). When $K > 2$, the upper bound on the minimum is $r_{min}$, the minimum correlation between the social indicators. (Proofs are shown in the appendix.)

Proposition 3 confirms the common intuition that agreement is maximized when people have the same weights on social indicators, and agreement is minimized when people have opposing (orthogonal) weights on social indicators. But intuition does not reveal the magnitude of the minimum $A_{ij} = r_{min}$. Note that the minimum correlation is not zero but may be higher or lower, depending on $r_{min}$. We show later that this is important in estimating agreement on many actual QOL indices.

The next proposition considers not just the minimum and maximum for $A_{ij}$ but the entire area where agreement is positive ($A_{ij} > 0$).

**Proposition 4:** When some correlations among social indicators $R_x$ are negative, then some persons in the group may disagree on the direction of the QOL index $(A_{ij} < 0)$. But the area where people agree appears to rise quickly toward 100 percent as $r_{min}$ rises toward zero. Specifically for the case of two social indicators ($K = 2$), even when $r$ is extremely negative ($r = -0.9$), over half of the area (59.8 percent) results in agreement on the direction of the QOL index.

To prove this proposition, one must first find the points where $A_{ij} = 0$. From equation (6), this is

$$A_{ij} = W_i^* R_x W_j^* = 0 \Rightarrow W_i R_x W_j = 0.$$
The last equality is true because, by equation (7) of the main text, $W_i^*$ is equal to $W_i$ divided by a constant. Hence, multiplying both sides by the constant retains the equality. Solving equation (9) in the general case is difficult, but the special case for $K = 2$ will be informative. When $K = 2$ component indicators, then the fact that weights must sum to 1 implies that $w_{i2} = 1 - w_{i1}$ and $w_{j2} = 1 - w_{j1}$. Making these substitutions in equation (9) yields

$$w_{i1}(w_{j1} + (1 - w_{j1})r) + (1 - w_{i1})(w_{j1}r + (1 - w_{j1})) = 0. \tag{10}$$

Then we can solve for $w_{i1}$ in terms of $w_{j1}$ and any $r$ as

$$w_{i1} = (w_{j1}(1 - r) - 1)/(2w_{j1}(1 - r) + r - 1). \tag{11}$$

This is a hyperbolic function in $w_{j1}$ and can be graphed for any choice of $[W_{i1}, W_{j1}]$ and for any value of $r$. Figure 1 graphs this function for all possible weights of person $i$ ($w_{i1}$) and person $j$ ($w_{j1}$) on the unit square, using the value of $r = -0.7$ for demonstration. The center region of the square (between the two hyperbolas) is the region where the two people agree on the direction of the QOL index ($A_{ij} > 0$), and the areas in the upper left and lower right are the regions where the two people disagree. Note that the diagonal line from [0, 0] to [1, 1] always results in perfect agreement between people because this represents the line where the two people agree on their weights. The region of agreement always spreads from this maximum on the diagonal line toward the minimum on the corners at [0, 1] and [1, 0]. Note also that the area in this graph where people agree is much larger than the area where people disagree. In this graph, the area where people agree corresponds to 74.6 percent of the total area (of all possible weights). The percentage of the area where people agree is a useful index because when people are distributed uniformly on the unit square, it predicts the actual percentage of people whose QOL indices will agree.

Table 2 calculates the area where QOL indices agree in the case of two social indicators, as a function of $r$. The top row shows that when the correlation between social indicators is $-0.90$ (very extreme), the proportion of the unit square where $A_{ij} > 0$ (minimal agreement exists on the direction of the index) is 59.8 percent. When the correlation between social indicators is $-0.80$, the percentage agreeing is higher at 66.3 percent. One can see that the percentage area increases rather quickly, so that when the correlation is $-0.5$, fully 84.6 percent of all possible weights result in agreement ($A_{ij} > 0$). If the distribution of weights in the population is uniform, then Table 2 also gives the proportion of people in the population who agree.
Note that even for extremely low $r$ (−.9), a majority of people still agrees on the direction of the QOL index.

We now consider whether researchers can construct a QOL index that will maximize agreement among individuals. Let $Z$ be any vector of weights that is a linear function of individuals’ weights $W_1, W_2, W_3, \ldots$. Then we prove (in the appendix) that the choice of $Z$ that maximizes agreement over all individuals is simply the mean weight vector across individuals, $W$.

**Proposition 5**: There is a unique weighting for any QOL index that maximizes the agreement $A$ with the index over all individuals $i$ in the population. This unique weighting for the QOL index is $W$, or the average weights (calculated over all individuals in the population).
Proposition 5 is helpful only if the distribution of weights is already known, as from a survey. If individuals’ importances are not known, then what weights should be used to create the QOL index to reduce the risk of disagreement most? This question defines a minimax estimator, which minimizes maximum possible disagreement. We can show that the equal-weighting policy $W_E = \frac{1}{K}, \frac{1}{K}, \frac{1}{K}, \ldots$ is the minimax estimator and therefore reduces the risk of disagreement most when weights are not known.

Proposition 6: When individuals’ weights are not known, then the unique weight $Z$ that minimizes maximum possible disagreement over all possible distributions is equal weighting: $W_E = [\frac{1}{K}, \frac{1}{K}, \frac{1}{K}, \ldots]^T$.

The proof is shown in the appendix. We note that equal weighting is a privileged strategy because, besides being the minimax estimator, it also minimizes disagreement under a Bayesian prior distribution of uniform weights. (This can easily be seen from Proposition 5 under the prior assumption that weights are uniformly distributed.) Hence, if no surveys have been done to estimate the importance that the population places on each attribute (a common occurrence), then equal weighting is optimal under both minimax and uniform prior assumptions. If surveys have been done to develop a better posterior distribution of the true weights, then in general, the mean weight vector $W$ in Proposition 5 will achieve better agreement for the QOL index than equal weighting.

### Table 2
Percentage of All Possible Weights for Two Social Indicators Where Two Individuals Agree on the Index ($A_{ij} > 0$), as a Function of Correlation $R_x$ Between the Social Indicators

<table>
<thead>
<tr>
<th>$R_x$</th>
<th>Percentage Where $A_{ij} &gt; 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>–.9</td>
<td>59.8</td>
</tr>
<tr>
<td>–.8</td>
<td>66.3</td>
</tr>
<tr>
<td>–.7</td>
<td>74.6</td>
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<tr>
<td>–.6</td>
<td>78.5</td>
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<tr>
<td>–.5</td>
<td>84.6</td>
</tr>
<tr>
<td>–.4</td>
<td>88.7</td>
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<tr>
<td>–.3</td>
<td>94.8</td>
</tr>
<tr>
<td>–.2</td>
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<tr>
<td>–.1</td>
<td>99.5</td>
</tr>
<tr>
<td>&gt; 0</td>
<td>100</td>
</tr>
</tbody>
</table>

Table 2 Percentage of All Possible Weights for Two Social Indicators Where Two Individuals Agree on the Index ($A_{ij} > 0$), as a Function of Correlation $R_x$ Between the Social Indicators

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The above propositions state generally how agreement $A_{ij}$ varies when individuals apply different weights to social indicators. We now analyze three specific examples to show how these results apply to actual QOL indices. The examples will show that, for many indicators, a very large majority will agree on QOL judgments. However, in other situations, a substantial possibility for disagreement on QOL indices exists.

Applications To QOL Indices

Example 1: Human Development Index 2001

Earlier, we described the HDI and its three component indicators. We also noted that the HDI weights these three indicators equally to derive the HDI index. How important is the equal-weighting assumption? How much disagreement would result if individuals apply different weights to the social indicators?

First, the correlations among the individual social indicators $R_{xx}$ must be computed. We computed these from the published data for the HDI 2001 for 162 nations, as shown in Table 3a. The correlations are all significantly different from zero and are quite high. These high correlations are consistent with previous findings on cross-sectional social indicators at the nation level (Morris 1979). What Morris (1979) did not comment on was that any resulting QOL index formed from these social indicators also would have high agreement among individuals.

To see this, we can use Proposition 3, which states that the minimum agreement will be $r_{\text{min}}$ in Table 3a, or $+.77$. Thus, we have the surprising conclusion that even people with diametrically opposed weights would have QOL indices that have correlation $r_{ij} = +.77$. The intuitive reason for this is that the underlying social indicators are near substitutes for each other. Hence, even people who disagree on the ethically appropriate weights can still agree on their QOL indices for the specific countries and time periods in question. The correlation of $+.77$ is within the common findings for test-retest reliability of a single measure. Hence, for the HDI, even worst-case weights will yield indices that are equivalent for most purposes.

Proposition 3 gives the minimum of $A_{ij}$ for the HDI 2001. But it is important to find the entire distribution of agreement among all pairs of individuals, to gauge overall agreement in the population. To estimate the distribution, Proposition 1 states that we must know not only the correlations among social indicators $R_x$ but also the distribution of individuals’ weights. We therefore examine several benchmark distributions of weights.
We first examine a uniform distribution. In a later section, we examine actual distributions of importance weights that are drawn from surveys.

To specify a first reasonable benchmark for distribution of weights in the population, we assume a uniform distribution, simulating 100 draws from a population whose importance weights are uniformly distributed along the unit interval [0, 1]. We used the method of Becker et al. (1987) to create random draws from this multivariate distribution, known as the Dirichlet distribution. The resulting distribution of $A_{ij}$ over all 4,950 possible pairs of the 100 individuals is shown in Figure 2. As predicted by Proposition 3, all correlations are positive, despite the fact that some individuals had diametrically opposed weights. As predicted by Proposition 3, the minimum $A_{ij}$ in the simulation is .82, above the theoretical minimum of .77. In fact, the distribution itself is more positive than predicted by the propositions because it is skewed toward the maximum of 1. Despite the intuition that the distribution of correlations among pairs of individuals on a QOL index composed from uniformly distributed weights might itself be uniform, the actual distribution is heavily skewed toward the maximum of 1. This is good news for agreement among individuals. The average correlation $A_{ij}$ in Figure 2 among people was +.97, with a standard deviation of .028. More than 93 percent of all possible pairs had correlations

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<th>(1) Log(GDP/capita)</th>
<th>(2) Life expectancy</th>
<th>(3) Education</th>
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<td>1</td>
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<table>
<thead>
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<th>(1) Log(GDP/capita)</th>
<th>(2) Life expectancy</th>
<th>(3) Education</th>
<th>(4) 1 – Gini index</th>
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<tr>
<td>.82</td>
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<td>1</td>
<td>1</td>
</tr>
<tr>
<td>.40</td>
<td>.40</td>
<td>.30</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 3: Correlations Among Social Indicators From (a) the Human Development Index (HDI) 2001 and (b) the HDI and Gini Coefficient
above +.90. This is far higher than many would expect, when weights are distributed uniformly.

In summary, our analysis of the effects of various weighting schemes for the HDI 2001 shows that the vast majority of possible weights (98 percent of the total volume) result in correlations between indices ($A_{ij}$) that are very high (greater than +.90). For the HDI 2001, different weights are simply not an impediment to agreement on a QOL index.
Example 2: GDP Per Capita and Income Equality

One of the reasons that weights do not matter in the HDI 2001 is that the underlying social indicators are highly correlated (e.g., the correlation between GDP per capita and health was .82). This is reasonable because all of the social indicators collected are meant to be measures of human development. When multiple measures of the same underlying construct are used, then we would expect them to have high correlations with each other. A more challenging example is the relationship between income equality and GDP per capita. These concepts are clearly different, and theorists have argued whether the direction of the relationship is positive or negative (see Firebaugh 1999 for a review). The United Nations Development Program (2001) reports, in a supplementary table, income equality measures for 111 nations—the largest number ever reported in a single source. We extracted the most common measure of inequality, the Gini coefficient of income distribution. Since the Gini coefficient varies from 0 (no inequality) to 1 (maximum inequality), we reversed its direction by using the transformation \((1 – \text{Gini})\). Hence, all importance weights remain in the positive quadrant. The correlations between equality \((1 – \text{Gini})\) and the three HDI indicators over the 111 nations are shown in Table 3b.

Note first that the intercorrelations among the three HDI indicators for the 111 nations (Table 3b) are quite similar to the ones computed over all 162 nations (Table 3a), and all correlations in the table are significantly different from zero. Note also that the simple correlation between GDP/capita and income equality is +.4. This figure is lower than those in the HDI, but it is significantly greater than zero and is consistent with multivariate results. How do different weightings affect a QOL index that includes not only HDI but also equality?

As in Example 1, a benchmark distribution of 100 random individuals with uniformly distributed weights was generated. The resulting distribution of \(A_{ij}\) of all 4,950 possible pairs of the 100 individuals is shown in Figure 3. Again, as predicted by Proposition 2, all correlations are positive, despite the fact that some individuals had diametrically opposed weights. As predicted by Proposition 3, the minimum \(A_{ij}\) in the simulation is .40, equal to the theoretical minimum of .40. Again, the distribution itself is more positive than predicted by the propositions because it is skewed toward the maximum of 1. The average correlation \(A_{ij}\) in Figure 3 among people is +.91, with a standard deviation of .01. More than 94 percent of all possible pairs had correlations above +.70 (the usual criterion for assessing good interrater reliability).
Proposition 6 predicts that we can generate even more agreement among individuals by constructing the equal-weights QOL index of \([.25, .25, .25, .25]\). The distribution of agreement between the equal-weight QOL index and the 100 simulated individuals is shown in Figure 4. As predicted by Proposition 6, average agreement increases. What was not
predicted was the skew toward 1, resulting in 93 percent of individuals with $A_{E,i} > .9$.

**Example 3: The Index of Social Health**

We previously described the ISH and its 16 component social indicators, and we noted that the indicators pertain to data for the United States for multiple years since 1970. To our knowledge, the correlations among these
indicators have not been published. Using the raw data from Miringoff and Miringoff (1999), correlations were computed and are shown in Table 4 for the 16 indicators. Note that, contrary to the previous cross-sectional examples, Table 4 displays many large negative correlations. For example, average weekly earnings are correlated at \(-.921\) with life expectancy at age 65 (while life expectancy has been increasing over time, weekly earnings of hourly workers have been declining). These large negative correlations provide the conditions for conflicting policy recommendations and for very low agreement among individuals on the resulting QOL index. Proposition 3 predicts that the lowest agreement among pairs of individuals will be \(r_{min}\) from Table 4, or \(-.94\). How much agreement would actually result from this QOL index with a population whose weights were uniformly distributed? Using the technique in Example 1 to generate individuals with uniformly distributed weights, we find levels of agreement that are surprisingly high. Average \(A_{ij}\) is \(+.40\), with a standard deviation of \(.45\), but again the distribution is strongly skewed toward 1, with fully 80 percent of the 4,095 paired comparisons resulting in \(A_{ij} > 0\) and 34 percent of paired comparisons with \(A_{ij} > .7\). The actual percentage of people who agree on the trend over time in the ISH \((A_{ij} > 0)\) will depend on the distribution of persons’ weights in the unit square. In particular, if weights themselves are negatively correlated, such that a person with a higher than average weight on \(W_1\) has a lower than average weight on \(W_2\), then the distribution would tend to the upper left and lower right sections of Figure 1, causing a decrease in the percentage of people agreeing on the index.

Proposition 6 predicts that the equal-weighting QOL should generate maximum agreement among uniformly distributed individuals. The distribution of agreement between the equal-weight QOL index and the 100 simulated individuals is shown in Figure 5. The mean \(A_{E,i}\) is \(+.67\) with a standard deviation of \(.39\). What was not predicted was the skew toward 1, resulting in 67 percent (a supermajority) of individuals with \(A_{E,i} > .7\) and 89 percent with \(A_{E,i} > 0\).

Summarizing the analyses of these three examples of QOL indices, two would experience very high levels of agreement \((A_{ij} > .7\) for a large majority of pairs) when weights are distributed randomly in the population. Hence, in these two cases, weights do not affect agreement very much (as predicted by Proposition 2) because the indices measure cross-sectional quality of life across nations, resulting in the consistently positive correlations among social indicators. Morris (1979) first pointed out that many social indicators are highly (and positively) correlated in a cross section, and his finding is reinforced here 20 years later with more countries. This
| (1) Wages (average weekly earnings) | 1.00 | -0.85 | -0.79 | 0.83 | -0.27 | -0.94 | -0.59 | -0.04 | -0.81 | -0.11 | 0.88 | 0.85 | 0.46 | 0.66 | 0.89 | 0.85 |
| (2) Life expectancy at age 65 | -0.85 | 1.00 | 0.83 | -0.89 | 0.61 | 0.93 | 0.72 | 0.27 | 0.92 | 0.43 | -0.91 | -0.66 | -0.67 | -0.83 | -0.94 | -0.89 |
| (3) 1 – percent dropouts from high school | -0.79 | 0.83 | 1.00 | -0.87 | 0.64 | 0.89 | 0.75 | 0.24 | 0.88 | 0.31 | -0.82 | -0.67 | -0.72 | -0.65 | -0.93 | -0.91 |
| (4) 1 – Gini | 0.83 | -0.89 | -0.87 | 1.00 | -0.68 | -0.94 | -0.82 | -0.28 | -0.82 | -0.48 | 0.80 | 0.69 | 0.74 | 0.64 | 0.94 | 0.91 |
| (5) Housing Affordability Index | -0.27 | 0.61 | 0.64 | -0.68 | 1.00 | 0.53 | 0.69 | 0.59 | 0.65 | 0.72 | -0.53 | 0.06 | -0.84 | -0.57 | -0.65 | -0.57 |
| (6) 1 – infant mortality rate | -0.94 | 0.93 | 0.89 | -0.84 | 0.53 | 1.00 | 0.80 | 0.23 | 0.87 | 0.30 | -0.87 | -0.77 | -0.62 | -0.67 | -0.98 | -0.95 |
| (7) 1 – percentage in poverty over 65 years | -0.59 | 0.72 | 0.75 | -0.82 | 0.69 | 0.80 | 1.00 | 0.47 | 0.73 | 0.34 | -0.56 | -0.46 | -0.68 | -0.43 | -0.84 | -0.83 |
| (8) 1 – unemployment percentage | -0.04 | 0.27 | 0.24 | -0.28 | 0.59 | 0.23 | 0.47 | 1.00 | 0.29 | 0.36 | -0.31 | 0.40 | -0.55 | -0.29 | -0.34 | -0.12 |
| (9) 1 – drug use rate of 12th graders | -0.81 | 0.92 | 0.88 | -0.82 | 0.65 | 0.87 | 0.73 | 0.29 | 1.00 | 0.31 | -0.91 | -0.63 | -0.69 | -0.84 | -0.93 | -0.84 |
| (10) 1 – traffic fatalities from alcohol | -0.11 | 0.43 | 0.31 | -0.48 | 0.72 | 0.30 | 0.34 | 0.36 | 0.31 | 1.00 | -0.31 | 0.06 | -0.61 | -0.54 | -0.35 | -0.37 |
| (11) 1 – violent crime rate | 0.88 | -0.91 | -0.82 | 0.80 | -0.53 | -0.87 | -0.56 | -0.31 | -0.91 | -0.31 | 1.00 | 0.59 | 0.64 | 0.88 | 0.89 | 0.76 |
| (12) 1 – percent children below poverty line | 0.85 | -0.66 | -0.67 | 0.69 | -0.06 | -0.77 | -0.46 | 0.40 | -0.63 | 0.06 | 0.59 | 1.00 | 0.20 | 0.40 | 0.69 | 0.80 |
| (13) 1 – suicide rate among 15-24 years | 0.46 | -0.67 | -0.72 | 0.74 | -0.84 | -0.62 | -0.68 | -0.55 | -0.69 | -0.61 | 0.64 | 0.20 | 1.00 | 0.66 | 0.73 | 0.63 |
| (14) 1 – birth rate to teenage mothers | 0.66 | -0.83 | -0.65 | 0.64 | -0.57 | -0.67 | -0.43 | -0.29 | -0.84 | -0.54 | 0.88 | 0.40 | 0.66 | 1.00 | 0.72 | 0.60 |
| (15) 1 – child abuse report rate | 0.89 | -0.94 | -0.93 | 0.94 | -0.65 | -0.98 | -0.84 | -0.34 | -0.93 | -0.35 | 0.89 | 0.69 | 0.73 | 0.72 | 1.00 | 0.94 |
| (16) 1 – percent covered by health insurance | 0.85 | -0.89 | -0.91 | 0.94 | -0.57 | -0.95 | -0.83 | -0.12 | -0.84 | -0.37 | 0.76 | 0.80 | 0.63 | 0.60 | 0.94 | 1.00 |
result tends to argue for a “single-factor” explanation of modernization, a conclusion also shared by the consortium of sociologists from the “Comparative Charting of Social Change” program (Langlois et al. 1994), who consider many more social indicators than we do.

But Morris’s (1979) conclusions referred only to social indicators in cross-sectional analyses, as in the first two examples. In contrast, the third
example is a time series of social indicators on a single nation. These type of data results in many negative correlations, which by Proposition 3 can result in much lower levels of citizen agreement (e.g., life expectancy above age 65 is negatively correlated with average weekly earnings in the United States since 1970). Hence, in the time-series (ISH) example, weights do matter, and it is possible to have severe disagreement, depending on the particular distribution of weights.

The reason why correlations tend to be more negative in time-series data than in cross-sectional data is due in part to “restriction of range” problems (e.g., life expectancy varied far less in the United States since 1970 than it does in a cross-sectional sample of nations, where Somalia has a life expectancy of only 40 years. We therefore predict that cross-sectional samples restricted to highly industrialized nations are likely to have more negative correlations than samples drawn from all nations, though this prediction must await future empirical test.) Another reason why correlations tend to be more negative in time-series data may also be due to preferences of individual nations. For example, the United States seems to prefer higher GDP/capita at the expense of some loss in equality, compared to European nations. Such a policy could result in negative correlation between these indicators as inequality is pushed up to gain GDP/capita. Whatever their cause, negative correlations tend to work against citizen agreement on QOL indices. This fact is unfortunate because national debates more often focus on time-series analyses (“Are you better off than 4 years ago?”) than on cross-sectional analyses (“Are we better off than Somalia?”). Yet even with large negative correlations from the third example, a QOL index can be constructed for individuals with uniformly distributed weights that allows a supermajority of 67 percent to endorse it with \( A_{E,i} > 0.7 \), with only 11 percent of individuals with \( A_{E,i} < 0 \).

The analysis so far has imposed few restrictions on people’s actual importance weights (simply that they are positive and, in some cases, that they are uniformly distributed). In the next section, we examine actual distributions of weights from individuals in the United States to test whether they conform to the conditions required for agreement on a QOL index.

Application to Sample Survey Data on Individuals’ Importance Weights for QOL Components

Two surveys of importance weights are examined here, both of which included international samples, though we consider only U.S. responses
here to examine agreement in a single country in a single language. The first is the World Values Survey (WVS; Inglehart 2000), which asks respondents in 50 countries to rate the importance of family, friends, leisure time, politics, work, and religion. The exact wording to the questions in 1995 was as follows: “Please say, for each of the following, how important it is in your life. Would you say xxx is very important (3), rather important (2), not very important (1), or not at all important (0)?” The scale is usually assumed to be equal interval (hence the codes are equal interval), and the anchoring at not at all important may be assumed to represent a weight of near zero. Consistent with our model, no negative weights are allowed. Table 5 shows the distribution of the six importance scales and their intercorrelations for the U.S. sample in 1995. They represent 1502 U.S. residents randomly selected and interviewed by telephone (39 of the original sample did not complete one or more of the ratings and were excluded). Note that the mean importance for family is highest, followed by friends, religion, work, leisure time, and politics. Inspection showed that all six distributions were single peaked and not bimodal. Furthermore, correlations are all significantly positive (with the exception of leisure with religion) but were all less than .25.

The four social indicators from Example 2 were reevaluated using the surveyed importances from a random sample of 500 respondents from the WVS. Agreement was much higher using actual weights from the WVS than from using a uniform distribution. The mean agreement $A_{ij}$ among the 124,750 pairs was +.99, with a standard deviation of .02. More than

<table>
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<tr>
<th></th>
<th>Family</th>
<th>Friends</th>
<th>Leisure Time</th>
<th>Politics</th>
<th>Work</th>
<th>Religion</th>
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<tr>
<td>Standard deviation</td>
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<td>0.70</td>
<td>0.88</td>
<td>0.90</td>
<td>0.87</td>
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Table 5
Means, Standard Deviations, and Correlations Among the Six Importance Scales From the World Values Survey ($n = 1,502$ U.S. Residents 1995 Only)
99 percent of all possible pairs had correlations above +.90. Similarly, the 16 social indicators from Example 3 were reevaluated using the surveyed importances of 500 randomly sampled WVS respondents. Again, agreement was much higher using actual weights. The mean agreement $A_{ij}$ among the 124,750 pairs was +.94, with a standard deviation of .07. More than 81 percent of all possible pairs had correlations above +.90. Mean agreement $A_{E,i}$ between the 1,502 respondents and the equal-weighting index was higher yet, at +.97 with a standard deviation of .04.

Why is agreement so high when using weights from the WVS? One important reason is that the distributions of weights were neither uniform nor bimodal, but all were strongly unimodal. Hence, instead of a uniform spread across the entire response scale, most people clustered near a single point on the response scale. To take the most extreme example, 95.1 percent of respondents said that family is very important. Even for the variable with highest standard deviation, 53.7 percent of respondents said that work is very important, and only 16 percent responded in the lowest two categories. Inspection of the weights for the other 40 countries in the WVS showed similar distributions and resulted in similar levels of agreement. Even when all countries were pooled, a random sample of 100 respondents yielded mean agreement $A_{ij}$ among the 4,950 pairs that was +.88, with a larger standard deviation of .11. More than 92 percent of all possible pairs had correlations above +.70.

A similar pattern of distribution was found in a second survey of importance weights by the Economist Intelligence Unit (EIU 2005). An online survey of current readers of The Economist magazine was conducted, with 3,160 readers from 147 countries providing complete responses. Respondents were asked to rate the importance of 10 social indicators on a 5-point scale, where 5 denoted very important and 1 denoted unimportant. This second survey has the disadvantage of not being a probability sample of voters in a nation but has the advantage of measuring more social indicators on a finer scale. The 10 importances measured were material well-being, your health, family relations, job security, job satisfaction, social and community activities, security situation in your country, degree of political and civil liberty in your society, and degree of social equality in your society. Table 6 shows the distribution of the 10 importance scales and their intercorrelations for the U.S. sample of 994. Note that the mean importance for “your health” is highest, followed by family relations, political and civil liberty, job satisfaction, job security, and security situation. Inspection showed that all six distributions were single peaked and not bimodal. The average correlation
### Table 6
Means, Standard Deviations, and Correlations Among the 10 Importance Scales From the Economist Intelligence Unit (n = 994 U.S. Residents Only)

<table>
<thead>
<tr>
<th>(1) Material well-being</th>
<th>(2) Your health</th>
<th>(3) Family relations</th>
<th>(4) Job security</th>
<th>(5) Job satisfaction</th>
<th>(6) Social and community activities</th>
<th>(7) Security situation in your country</th>
<th>(8) Degree of political and civil liberty in your country</th>
<th>(9) Degree of gender equality in your society</th>
<th>(10) Degree of social equality in your society</th>
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</thead>
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Mean: 3.50
Standard deviation: 0.88

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between importances was rather low at +.20, but the largest negative correlation was even smaller at −.07 (between social equality and material well-being). The only other remarkable correlations were a cluster among (8) political and civil liberty, (9) gender equality, and (10) social equality, all of which were positive.
The 16 social indicators from Example 3 were reevaluated using the surveyed importances from 500 randomly selected EIU respondents. As with the WVS, agreement was much higher using actual weights from the EIU compared to using uniformly distributed weights. The mean agreement $A_{ij}$ among the 124,750 pairs was +.93, with a standard deviation of .09. More than 80 percent of all possible pairs had correlations above +.90. And again, mean agreement $A_{E,i}$ between the 994 respondents and the equal-weighting index was higher yet, at +.97. Figure 6 shows the distribution of agreement for the 994 respondents with the equal-weighting index. In summary, both surveys predict that a supermajority of citizens in the United States would endorse a QOL index with these 16 social indicators.

**Discussion and Conclusions**

Of the many QOL indices that have been proposed to date, none have explicitly considered whether individuals would agree with their choice of indicators and weights. This article proposes a simple model for predicting the extent of individuals’ agreement on QOL judgments with other individuals and investigates whether it is possible to create a QOL index from real social indicators that will be endorsed by a majority of individuals. In every case we examined, using both real surveys of individuals’ importance weights as well as a more general uniform distribution, it was possible to create a QOL index that a majority of individuals endorse (i.e., they agree at least with the direction of the QOL index). Specifically,

1. When correlations among social indicators are all positive (as in all cross-sectional data sets considered here), then agreement will be high regardless of the variation in weights. This highlights the paradoxical result that people may argue in theory about whose weights are more ethically appropriate, but in practice, their conflicting weights will yield substantial agreement on the overall QOL index. This result is well known in regression analysis but has not been observed in the context of social indicators.
2. When some correlations among social indicators are negative (as in time-series data sets, where trends diverge for some indicators), intuition suggests and Johansson (2002) predicts that chances for agreement are slim. However, our results are the first to show that disagreement is much rarer than expected and occurs only when the distribution of individuals’ weights is (a) bimodal and (b) negatively correlated (i.e., when individuals’ weights are diametrically opposed). These conditions did not occur in the surveys of real importance weights or in the more general uniform distributions, with
the result that agreement on the QOL indices was much higher than expected from simple intuition or from previous work. The reason that the uniform distribution generates such high agreement is because it is not bimodal. It contains a broad “middle segment” in the center whose weights are near enough to each other to generate agreement at the average W. The surveys of real weights are very strongly unimodal and so generate even higher agreement. However, highly polarized and emotional issues such as abortion are more likely to show bimodal weight distributions, generating insufficient agreement for a majority to endorse.16

3. We also have shown that researchers can increase the level of agreement for a QOL index by weighting the components appropriately. Agreement is maximized by using the average weights from a survey of individuals’ importances. Alternatively, if no surveys exist, equal weighting of indicators is the minimax estimator that minimizes disagreement even among diametrically opposed individuals. Note that in current practice, many QOL indices already use equal weighting of indicators, though their authors admit that they do not know whether this weighting is correct. The current results can now place current practice on a sound theoretical footing and show how it is possible to further increase agreement through surveys.

**Implications for QOL indices.** Our results predict high agreement among QOL indices that are constructed according to the assumptions in equation (2). These assumptions are as follows: (a) all individuals place positive weights on each attribute, and (b) all individuals use general additive models to judge QOL. With respect to the first assumption, many existing QOL indices already conform. For example, everyone prefers more longevity, higher income, and more education (all other things being equal) in the Human Development Index, and hence the positivity requirement is met. Another conforming survey is Inglehart’s (2000) longitudinal study of values because the WVS allows only nonnegative weights.

However, there are indices that fail the positive weights assumption. For example, *Money* magazine’s index of “Best Places to Live” includes an indicator “average price of a 3-bedroom home.” Some people (homeowners) would place a high positive weight on this, but others (homebuyers) would place a high negative weight, violating equation (2). In fact, this is an example of a zero-sum negotiation game where every gain for a buyer is a loss for the seller, and joint gains are always zero regardless of the price. *Money* magazine probably included this indicator because their readers are primarily home buyers, but this indicator is not suitable for a QOL index because (a) QOL does not change with this indicator since the joint sum is always zero, and (b) sharp disagreement would result because
equation (2) is violated. Negotiation researchers (Pruitt and Kim 2004; Carnevale and Pruitt 1992) recommend instead including indicators that allow positive joint gains to enhance the framing of shared interests. Much research has shown that this increases the likelihood of agreement and increases joint gains in negotiations. Applying these principles to the Money magazine example, a simple “laddering” procedure (“what deeper goals are you trying to achieve with lower housing prices/higher housing prices?”) could replace the single zero-sum attribute (price) with two shared goals: lower cost per square foot of new construction and higher personal income. Both of these new indicators would conform to our assumptions and would result in higher likelihood of agreement.

This example points out that not all social indicators are appropriate in QOL indices, and inclusion should be contingent on each indicator’s (a) reliability, (b) perceived importance by citizens, and (c) likelihood of agreement on the resulting QOL index, as derived here. Another important example of indicators to exclude from QOL indices is tax policy, because conservatives place a negative weight on average tax burden and liberals tend to place a positive weight. Tax policy is better viewed as a means to an end, and a successful QOL index would again apply laddering to include the end-state variables (e.g., better health care, education, pollution control, and economic growth). These examples show that a QOL index would not remove the need for policy analysis and political discussion, but it would focus policy analysis and politics by forcing proponents to estimate each policy’s results on the QOL index.

The second assumption from equation (2) is that individuals use a simple additive model to form judgments about QOL. While this model is confirmed by Sastre’s (1999) study of how individuals evaluate well-being and by Srinivasan and Park’s (1997) results predicting product preferences, it needs more empirical research. In particular, substitutability or complementarity may exist between social indicators that would require modeling interactions among indicators. For example, an individual with higher average income may consider life expectancy more important than an individual with very low income (as life becomes more “worth living,” longer life may be more valuable). Such complementarity could be added to equation (2) by constructing an interaction term, although its importance weight would be more difficult to measure in surveys. Empirical tests for these interactions could be done by surveying individuals and determining their preferences for hypothetical “bundles” of social indicators for their social unit. To our knowledge, no such studies have been
done for representative samples of any social units. Such work would be invaluable for constructing a QOL index that correctly mirrors the preferences of the social unit.\(^{17}\) The methods we outline here also allow deeper analysis of the more than 20 QOL indices that have been proposed. None of them agrees perfectly with each other, and some disagree even in direction with others. Our analysis in equation (6) now allows researchers to “decompose” the sources of disagreement into those due to selection of different (though correlated) indicators \(R_x\), those due to use of different weights to construct the indicator \(W_z\), and those due to different importance weights in the target population \(W_i\).

Our conclusions must be viewed with caution for several reasons. First, we made use of existing surveys of individuals’ weights that were not specifically designed to measure weights for the QOL indexes reviewed here. Most important, the weights in equation (2) must be correct to a ratio scale (because the zero point is meaningful), whereas the Likert scales in the WVS are often considered correct only to an interval scale. However, the particular anchoring in WVS (not at all important = 0) appears to assign the appropriate response to the zero point, and validation studies of equation (2) in choice surveys (Srinivasan and Park 1997) show that this type of scale predicts preferences quite successfully. Another limitation of the WVS survey is that it contained only six general importance weights (family, work, etc.) measured on a scale with only 4 points. However, the finer gradations available with a 10-point scale are unlikely to change our results. We show that agreement is most likely when (a) weight distributions are all unimodal rather than uniform or bimodal, (b) correlations are mild and positive, and (c) few people use the zero point of the scale. All three of these conditions are true in the surveys we examined, and it seems unlikely that an expanded rating scale or a different zero point would change these properties.

Throughout this article, we have assumed that individuals are members of a political state, but our results can be directly generalized to expert committees, such as a task force of analysts attempting to agree on the effectiveness of a mix of government policies. Then the prospects for finding agreement on the correct mix of policies are given by the above propositions (assuming that the committee members reveal in good faith their beliefs about the effectiveness of each policy).

Researchers have debated the appropriateness of forming summary indices of social well-being for years. But they have investigated only extreme cases that predict high levels of disagreement among individuals
with differing weights. In contrast, we examine the entire range of possible conditions and then study the resulting agreement among individuals for several real social indices. In every case, substantial agreement exists and is much higher than expected by intuition. In every case, a QOL index could be constructed that a large majority of individuals would endorse because they would agree when the QOL index is rising and when it is declining—of prime importance for policy makers.

Appendix

The following are proofs of propositions not given in the main text.

Proof of Proposition 3

To prove the location of the maximum and minimum, one can compute the derivatives of $A_{ij}$ with respect to weights $W_i$ and $W_j$. Taking the derivative of equation (6) in the main text, subject to the constraint that the weights are standardized to 1 ($W_i^T R W_i)^{1/2} = 1$), gives the set of Lagrangian equations:

$$
\begin{align*}
\frac{dA}{dW_i} &= RW_j - 2\lambda_1 RW_i = 0, \\
\frac{dA}{dW_j} &= (W_i^T R)^T - 2\lambda_2 RW_j = 0, \\
\frac{dA}{d\lambda_1} &= W_i^T RW_i - 1 = 0, \\
\frac{dA}{d\lambda_2} &= W_j^T RW_j - 1 = 0.
\end{align*}
$$

Combining the first two equations in equation (A1) gives the condition for the optimum as $W_i = 2\lambda_2 W_j$, or that $W_i$ must be proportional to $W_j$. The third and fourth equations require that both $W_i$ and $W_j$ be standardized to unit length. Hence, $W_i$ must not only be proportional to $W_j$ but must be equal so that $\lambda_2 = 1/2$. Substituting $W_i = W_j$ in equation (6) of the main text shows that this point is a maximum and that $A_{ij} = 1$ there. This completes the proof for the maximum.

Since the only interior optimum in equation (6) is a maximum, then the minimum $A_{ij}$ must be found at the extreme points of the constrained function. In general, the extreme points lie along all edges of the unit hypercube $[w_1, w_2, w_3, \ldots]$ such that all $w_i > 0$ and $\sum_i w_i = 1$. To search all of these points would require an extensive program with nonlinear constraints. However, the minimum for the special case when $K = 2$ is easy to calculate, and it provides a good approximation to the minimum agreement for $K > 2$ in later examples.

The extreme points for the case when $K = 2$ are just $W_i = \{0, 1\}$, $W_j = \{0, 1\}$. Evaluating $A_{ij}$ at each of these points reveals a minimum at $W_i = [1, 0]^T$ and $W_j = [0, 1]^T$, with $A_{ij} = r$ at that point. Generalizing to $K > 2$, boundary conditions will always occur at the vertices of the unit hypercube, where $W_i$ places all
weight on one social indicator (which we label indicator $m$), and $W_j$ places all
weight on a different social indicator (which we label $n$), such that $W_i$ is ortho-
gonal to $W_j$. Evaluating $A_{ij}$ at the vertex where $W_i$ places all weight on the indicator
$m$ and $W_j$ places all weight on indicator $n$ yields $A_{ij} = r_{mn}$. The minimum of these
vertices is simply $r_{min}$, the minimum correlation among social indicators. We cau-
tion that $r_{min}$ is only an upper bound on the global minimum because even smaller
values of $A_{ij}$ might be found by evaluating the edges rather than just the vertices
of the unit hypercube, where individuals may place nonzero weights on several
social indicators. But analysis of the examples later shows that $r_{min}$ is a good
approximation to the overall minimum $A_{ij}$.

**Proof of Proposition 5**

Compute the sum of squared errors between $Q_Z$ (the QOL scores of an arbitrary
weighting vector $Z$) and $Q_i$ over all $I$ individuals:

$$SSE = \sum_i (Q_Z - Q_i)^T (Q_Z - Q_i).$$

(A2)

From equation (2) of the main text, this expands to

$$SSE = \sum_i (XZ - XW_i)^T (XZ - XW_i),$$

(A3)

which, by transposing, simplifies to

$$SSE = \sum_i ((Z - W_i)^T X^T) X (Z - W_i) = \sum_i (Z - W_i)^T R_x (Z - W_i).$$

(A4)

Note that equation (A4) is a quadratic form with $R_x$ symmetric. The minimum
SSE can then be found as the point at which the derivative of SSE with respect to
$Z$ is zero:

$$dSSE/dZ = 0 = \sum_i 2R_x (Z - W_i).$$

(A5)

Since both sides can be multiplied by $(2R_x)^{-1}$ with no change, this simplifies to

$$0 = \sum_i (Z - W_i) \Rightarrow Z = \sum_i W_i/I = W.$$

(A6)

This proves that $W$ is the unique weighting vector for a QOL index that minimizes
the sum of squared errors $SSE_{Zi}$ between $Q_Z$ and $Q_i$, where $i$ ranges from 1 to $I$
across all individuals in the population. Following the usual results from regression
that minimizing $SSE_{Zi}$ is equivalent to maximizing the correlation coefficient $A_{Zi}$,
this completes the proof.

**Proof of Proposition 6**

To obtain the minimax estimator, find the estimator $Z$ that minimizes the max-
imum disagreement among all possible $W$s:
\[
\begin{align*}
\frac{\partial A}{\partial W_i} &= RW_j - 2\lambda_1 RW_i = 0 \\
\frac{\partial A}{\partial W_j} &= (W_i^T R)^T - 2\lambda_2 RW_j = 0 \\
\frac{\partial A}{\partial \lambda_1} &= W_i^T RW - 1 = 0 \\
\frac{\partial A}{\partial \lambda_2} &= W_j^T RW - 1 = 0
\end{align*}
\]

(A7)

From Proposition 3, the inner maximization (maximum disagreement) occurs when individuals’ weights are diametrically opposed—that is, when the weights lie at the vertices of the unit hypercube, where \( W_i \) places all weight on one social indicator (which we label indicator \( m \)), and \( W_j \) places all weight on a different social indicator (which we label \( n \)), such that \( W_i \) is orthogonal to \( W_j \). For an arbitrary number of individuals \( I \), maximum disagreement occurs when each group gives all their weight to one of the \( K \) indicators and ignores all others:

\[
W_i = [1, 0, 0, \ldots, 0]^T \text{ for } [1 \leq i \leq I/K]
\]

\[
= [0, 1, 0, 0, \ldots, 0]^T \text{ for } [I/K + 1 \leq i \leq 2I/K] \ldots
\]

(A8)

For \( I(K - 1)/K + 1 \leq i \leq I \].

Now examining the outer minimization in (A6), Proposition 5 shows that this minimum exists for any \( W_i \), and the weights \( Z \) that achieve this minimization are just \( Z = \sum_i W_i / I \). Substituting (A8) into this yields the minimax estimator: \( Z = \sum_i W_i / I = [I/K, I/K, I/K, \ldots, I/K] = [1/K, 1/K, 1/K, \ldots] \). This is simply the equal-weighting policy.

Notes

1. For a review of recent developments in the field of social indicators, see Land (2000); for a statement on the uses of social indicators, see Ferriss (1988).


3. Throughout the text, weights will refer to “importance weights.”

4. See also Hagerty et al. (2001) for a review and evaluation of the Human Development Index (HDI).

5. To ensure that no indicator will dominate, these indices adjust the range of each indicator by dividing by the standard deviation or the range of each.

6. The problem of selecting the component indicators that comprise a QOL index is a perennial one where ideal procedures often crash against the reality of the data available for comparisons among social units in the cross section and/or over time. Indeed, the selection of component indicators all too often has been arbitrary and not justified on the basis of a theoretical conceptualization and/or prior research evidence (see, e.g., Booyens 2002 for a discussion of this problem in the context of composite indices of development, such as the HDI). Since the pioneering work of Andrews and Withey (1976) and Campbell, Converse, and Rodgers (1976) more than 25 years ago, however, there have accumulated numerous social
psychological studies of the components and determinants of subjective well-being, life satisfaction, and happiness. The results of reviews and syntheses of these various studies (e.g., Cummins 1996, 1997) now can be used to inform the selection of the components of summary well-being indices (as, e.g., in the work of Land et al. 2001 on a summary index of child and youth well-being). In this way, while the constraints of available data always will force compromises, the evidence from studies of what leads to individual subjective well-being, life satisfaction, and happiness can be used as an empirical basis to guide the selection of component indicators.

7. Additional observations on the implications of the positive weights assumption are given below in the Discussion and Conclusions section.

8. Alternative measures of association are Spearman’s rank correlation coefficient and Kendall’s coefficient of concordance (Hollander and Wolfe 1973). Both of these are restricted to rank-order properties and cannot easily incorporate importance weights as we do. In addition, while the qualitative properties of the model we specify below generally would apply to these alternative correlation coefficients, the algebra would become hopelessly complex and make difficult, if not impossible, the derivation of the results we obtain.

9. The use of the Pearson coefficient to measure agreement between individuals’ importance weights is appropriate even in the case that the weights are measured by the conventional 1 to 3 (or 5) rating scales of sample surveys—as long as one seeks only to draw conclusions about the measurements (i.e., the 1 to 3 [or 5] ratings) themselves (Sarle 1995). For example, if we want to test the hypothesis that the mean importance weights of two component indicators are equal, then we need not be further concerned about measurement models. If, however, we want to draw conclusions about the underlying latent dimension of importance of the component indicators to the individuals surveyed, then we either must use a measurement procedure for the importance weights (such as conjoint measurement; see Krantz et al. 1971) that gives interval-scale properties to the measured importance weights or use a measurement model such as a Rasch model (Arminger, Clogg, and Cheng 2000) that relates the measured weight scores to the latent dimension in a possibly nonlinear way and thus produces nonequal intervals among the measured weights. To date, there are no studies of the relative importance of component indicators of QOL composite scores that use anything other than the standard rating scales of sample surveys. Accordingly, the model and analyses we present can be regarded as pertaining to the properties of these weights, viewed as measurements themselves and as approximations to the individuals’ underlying latent dimension of importance of the component indicators that may not take into account possible nonlinear relationships to the underlying dimension.

10. The calculation of $A_{ij}$ is analogous to a rotation of axes, where the original axes are not orthogonal. In the usual case where the original axes are orthogonal, the normalization is simply the sum of squares of the weights $W_i$. But here, because the original axes are not orthogonal, the cross-products are not zero and must be included in the computation of $W_i^*$. 

11. More formally, $W_i^* R_i W_i$ in equation (7) is always nonnegative because $R_i$ is positive definite in a quadratic form.

12. The subsample of 500 was randomly selected from the full sample of 1,502 using the SPSS “select” function. The subsampling was necessary because all 124,750 possible pairs in the subsample were examined. Examining all 1,127,251 possible pairs in the full sample was beyond the capacity of our personal computers and was unnecessary because our subsample was representative of the full sample. The exact pairing for the analysis reported was GDP/capita with the importance of work, life expectancy with the importance of family, education...
with the importance of leisure time, and Gini with the importance of politics. This pairing is far from ideal, but surveys assessing importance of these four indicators do not exist. Sensitivity analysis showed that alternate pairing yielded very similar overall agreement.

13. The 16 indicators were paired with the six importance ratings as follows: The importance of work weighted indicators (1) wages, (5) housing affordability, and (8) employment. The importance of family weighted indicators (2) life expectancy, (6) infant mortality, (14) birth rate to teenage mothers, (12) children below the poverty line, (15) child abuse, and (16) percentage covered by health insurance. The importance of politics weighted indicators (4) Gini and (7) poverty rate over 65. Ideally, a survey should directly assess the importance of each of the 16 indicators for each respondent. But that survey does not exist. Sensitivity analysis (available from the authors on request) showed that overall agreement changed very little when alternate importances were assigned to the indicators.

14. The 16 indicators were paired with the 10 importance ratings as follows: Importance for material well-being was used as the weight for the social indicators (1) wages and (5) housing affordability. Importance of “your health” was used to weight (6) infant mortality, (9) drug use of 12th graders, (10) traffic fatalities resulting from alcoholism, (13) suicide rate, and (16) health insurance. Importance of family relations was used to weight (14) birth rate to teenage mothers, importance of job security was used to weight (8) unemployment, and importance of job satisfaction was used to weight (3) dropouts from high school. Importance of “security situation in your country” was used to weight (11) violent crime and (15) child abuse. Finally, importance of social equality was used to weight (7) poverty rate over 65 and (12) children below the poverty line.

15. Johansson (2002) argues that weights collected from many surveys are suspect because individuals have not devoted much thought to the trade-offs and require further education in the form of “town meetings” and education by experts. We therefore attempted to survey a population that has devoted their lives to education and to research on the trade-offs and interactions among social indicators—sociologists themselves. A convenience sample of 26 professional sociologists at an international conference in Europe completed the same questions as in the World Values Survey (WVS). The resulting distributions appeared similar to those of WVS respondents in that all distributions were unimodal, and correlations among weights were mild and close to zero. Actual weights from all 325 possible pairs of 26 sociologists yielded a mean of .98 with a standard deviation of .02. Ninety-five percent of the correlations were above .94, and the minimum was .88.

16. Gallup (2005) found that 40 percent of people in the United States believe that abortion is morally acceptable, 51 percent that it is morally wrong, while 9 percent said it depends on the situation or had no opinion.

17. Money magazine now surveys a representative sample in the United States for ratings of importance for various indicators in its “Best Places to Live” index. However, the magazine performs no tests for possible interactions and omits many indicators altogether.

References


**Michael R. Hagerty** is a professor of marketing at the Graduate School of Management, University of California at Davis. His articles have appeared in the *Journal of Personality and Social Psychology, Psychometrika, Marketing Science, Journal of Marketing Research, International Journal of Forecasting, Social Indicators Research*, and *Journal of Consumer Research*. He is a fellow of the International Society for Quality of Life Studies and won the Best Research Paper Award for 2003 in the *Journal of Happiness Studies*.

**Kenneth C. Land** is the John Franklin Crowell Professor of Sociology and director of the Center for Demographic Studies at Duke University. His research interests include the development of mathematical and statistical models and methods for substantive applications in demography, criminology, and social indicators/quality-of-life studies. He is an elected fellow of the American Statistical Association, the Sociological Research Association, the American Association for the Advancement of Science, the American Society of Criminology, and the International Society for Quality of Life Studies.