

Classifying Personal Networks using Latent Roles

Abstract. An individual's personal network encodes his/her social contacts as well as the relations among these. Personal networks are considered to be characterizing and meaningful variables of individuals—supplementing more traditional characteristics such as age, gender, race, or job position. In this paper we present a highly flexible method for exploration and analysis of an *ensemble* of personal networks. We define similarities and dissimilarities among networks by considering *who is in the network* and *how they are connected*. These similarities can be used to cluster a network ensemble and, thus, detecting typical forms of personal networks. Our method is illustrated on a collection of hundreds of personal networks of migrants.

1 Introduction

In social science individuals are often characterized and distinguished by personal attributes such as age, gender, race, job position, or income. Additional meaningful information is given by the individual’s personal network, i. e., his/her social contacts and relations among them. For instance, it has been shown that the structure of personal networks correlates with psychological indicators [Kalish and Robins, 2006]. Likewise, personal networks have been used to define user roles in Usenet newsgroups [Welser et al., 2007] and to characterize the acculturation of migrants [Brandes et al., 2008], [Molina et al., 2008]. Note that in these applications one has to analyze and compare a *set* of networks rather than a single instance (see, e. g., [Faust and Skvoretz, 2002], [Butts and Carley, 2005], [Faust, 2006]). In this paper we call a set of networks that stem from the same underlying process, such as repeated data collection by using the same questionnaire, a *network ensemble*, compare [Brandes et al., 2009].

Clearly, the networks in a given ensemble can be compared and characterized based on any global structural property, such as density, clustering coefficient, or degree distributions. In this paper we compare networks based on two aspects: (1) “who is in the network”, i. e., what are the individual characteristics of the *alters* in the network and (2) “how are they connected”, i. e., what is the structure of ties among the alters.

2 Empirical Dataset

To show an example of how our method is applied to real data we chose a number of personal networks collected from immigrants in Spain and the USA with the help of the EgoNet¹ software. In the following we will describe the underlying data set that will later be used for an example analysis of this data that sketches the possibilities of our proposed method. Each of the 504 networks describes the social surrounding of a migrant to Spain or the USA, originating from an South-American, Middle-American or African country. Since the underlying data is equal to the data set used in [Brandes et al., 2008], we reproduce that data description. Each respondent was asked to provide the following four types of information:

1. **(questions about ego)** 70 questions about the respondent herself, including age, skin color, years of residence, questions from traditional acculturation scales and health related questions
2. **(name generator)** A list of 45 persons (referred to as *alters*) personally known to the respondent. The alters are the nodes in the respondent’s personal network.
3. **(questions about alters)** 12 questions about each of the 45 alters, including country of origin, country of residence, skin color, and type of relation to ego.
4. **(ties between alters)** For each of the 990 undirected pairs of alters, the evaluation whether they know each other. The three possible choices were

“very likely”, “maybe” or “unlikely” and we introduced an edge in the network only if the respondent chose “very likely”.

This seems from a statistical point of view like a very small data set for each of the respondents. For our method it offers the opportunity to show the applicability on this kind of data. Through the use of attributes and the combined analysis of the complete ensemble of networks we are still able to extract some qualified statements based on clustering methods.

3 Abstract ensemble descriptions and how to obtain them

In this part we describe the model used to abstract an ensemble of networks and a method to extract such a model from a given ensemble. Our model has similarities to block models (c.f. [Brandes and Erlebach, 2005] or [Wasserman and Faust, 1994]) for graphs used in other approaches for abstractions of single graphs. In block models the focus of examination is the relational structure within the graph and the target is a model that groups the nodes into positions such that nodes of one position have neighbors of equal positions. In contrast to this, our model is based on positions characterized by attributes of nodes and relations between these positions described by the number of links between members of the different groups. Further our intention is not only to describe a single network, but to characterize a whole collection of networks.

We call an abstraction as described here a *relaxed role graph*, analog to block models we call the nodes of the role graph *positions*. Since only relaxed role graphs are considered in the remainder we will freely call them role graphs for shortness. The expected input of our method is a collection of networks, with nodes labeled by a number of arbitrary attributes. After a preprocessing step in which the attributes are converted to vectors in \mathbb{R}^d , we group the nodes of the ensemble (the nodes of networks) according to similarities of these vectors. This partition of the nodes of the ensemble is used as a basis for building a role graph. In the remainder of this chapter a more detailed description of our model is given, followed by our proposal on how to obtain such a model from a given ensemble. A possible, and in our application used, method to transform attributes of certain types into vectors of real numbers is given in section 4. For the moment we assume that the attributes of the nodes are given as or transformed into vectors of real numbers.

3.1 A model for simplifying structural descriptions

Consider a collection of networks $(V_i, E_i), i \in \{1, \dots, N\}$ with $V_i = \{v_1^i, \dots, v_{n_i}^i\}$, v_j^i being actors represented by their attribute vectors $\mathbf{v}_j^i \in \mathbb{R}^d$ and E_i being the set of undirected links between the nodes of V_i , described as sets of the connected nodes. In the remainder we will identify v_j^i with the corresponding attribute vector \mathbf{v}_j^i for clarity of description. A relaxed role graph of such a

collection of graphs is a tuple (P, A) where $P = \{p_1, \dots, p_r\}$ is the set of positions and analog to the nodes, positions p_i are identified with vectors $\mathbf{p}_i \in \mathbb{R}^d$. A is a real, symmetric $r \times r$ matrix with $A_{i,j} \in \mathbb{R}$ giving edge weights between actors of the positions i and j . This model is intended to abstractly describe an ensemble of networks in the following way: The positions describe typical nodes of the network, that is in the set of all nodes of the network clusters of nodes appear such that the members of the cluster have small pairwise distances, while they share considerably higher distances to nodes of other clusters. The edge probabilities are intended to describe the typical relation structure between nodes belonging to different positions. Let $F \subseteq V_i$ be the set of nodes of a network (V_i, E_i) assigned to the position p_f and analog is $G \subseteq V_i$ the subset assigned to position p_g . Then in the ideal case, the weight derived from the number of edges between the nodes of F and nodes of G approximates the weight given in the role graph by $A_{f,g}$:

$$A_{f,g} \approx \frac{e(F,G)}{\sqrt{|F||G|}},$$

where $e(F,G) = |\{\{f,g\} \in E_i : f \in F \wedge g \in G\}|$ is the number of existing edges connecting nodes in F to nodes in G . We follow the idea from [Brandes et al., 2008] that $A_{f,g}$ denotes the geometric mean of the expected number of neighbors in the opposite position for the nodes of the two positions. Therefore we refer to the derived edge weights as *average degree* in the remainder.

3.2 Overview of the method

Since our method has many parts, we give a short overview of what the single steps are and what their intention is, in parenthesis we give the corresponding section where the step is described in more detail.

1. choose the focus of examination by selecting weights for the attributes (3.3.1)
2. cluster the nodes of the ensemble to find typical network positions (3.3.2)
3. extract a relaxed role graph from these positions and the assignments of the nodes to the positions (3.3.3)
4. for each network, compute a description of how exactly it is described by the role graph (3.3.4)
5. cluster the set of networks in the ensemble using the obtained descriptions (3.3.5)
6. for each of the obtained clusters extract a role graph specifically describing the networks of that cluster (apply steps 2 and 3 recursively to the individual clusters)

3.3 Detailed method description

In this section we describe in detail the algorithm that extracts a model description as presented above for a given ensemble.

3.3.1 A focus adapting distance measure The possibility to control the influence of the different attributes on the clustering process of the nodes is one of the major aspects when the method is applied to real data. Therefore we will show a method to achieve this. As a basic distance between two nodes, respectively their attribute vectors we use the l_1 -norm of the vector difference: $d(\mathbf{u}, \mathbf{v}) = \sum_{i=1}^d |\mathbf{u}_i - \mathbf{v}_i|$, where \mathbf{u} and \mathbf{v} are the attribute vectors of the nodes and d is the number of dimensions needed to express the node attributes. This distance is linear depending on the distances between the attribute values of the two nodes and the individual distances sum up independently from each other, as opposed to e.g. an angle based distance.

As mentioned before, the importance of an attribute can be expressed as a weight. Given a weight vector \mathbf{w} that contains weights $w_a \geq 0$ for all attributes a we can modify the distance as follows: $d_W(\mathbf{u}, \mathbf{v}) = \sum_{i=1}^d \mathbf{w}_{m(i)} |\mathbf{u}_i - \mathbf{v}_i|$, where $w_{m(i)}$ is the weight of the attribute that is encoded by coordinate i of the attribute vector. This is equal to a scaling of the vector space the attributes are mapped to. By enlarging certain dimensions the corresponding attributes will gain more influence on the overall distance.

3.3.2 Determination of role graph positions To determine positions for a role graph description we concentrate on the demand that nodes of one position should have homogeneous attributes. Further the positions should have significant differences in the attributes of their nodes which lead to a clustering problem. This can be solved by any of the standard algorithms independently from the general method. The outcome of this clustering is to a large extend depending on the used distance measure. Therefore the distance measure proposed in section 3.3.1 offers the possibility to concentrate similarities within the searched clusters on combinations of individual attributes.

3.3.3 Model extraction The outcome of this first step is a partition $\mathcal{P} = \{P_i : i = 1, \dots, r\}$ of the set of all nodes in the ensemble such that each node belongs to exactly one partition: $\bigsqcup_{P \in \mathcal{P}} P = \bigcup_{i=1}^N V_i$. This implies for each network an assignment of the nodes to positions of the role graph. The partition further implies the positions of the role graph as the average of the vectors in each part:

$$\mathbf{p}_i = \frac{1}{|P_i|} \sum_{\mathbf{v} \in P_i} \mathbf{v}.$$

Computing edge weights Since in the first step all the role graph positions and the assignments of nodes to these positions are fixed, the average degrees of the role graph can be directly derived from the union of all networks.

$$A_{i,j} = \frac{1}{\sqrt{|P_i||P_j|}} \sum_{k=1}^N |\{\{a,b\} \in E_k : a \in P_i \wedge b \in P_j\}|$$

As in the model description, the number of existing edges between nodes of the different positions are normalized to the geometric mean of the expected number of neighbors in the opposing position.

3.3.4 Using role graph projections for network description The general purpose of our method is not only to give one abstract description of the whole ensemble, but also to characterize structural differences between the networks of the ensemble. In this part we will exploit the projections of the individual networks to the role graph for a clustering of these networks. The result is expected to highlight the structural trends that appear within the ensemble.

Again this task can be transformed to a clustering of vectors with real valued components. For that purpose we will describe each network with a *matching vector* that is derived from the projection of the network to the role graph. For a single network this vector consists of two blocks. The first block describes the matching quality of the nodes of the network compared to the positions they are assigned to and the number of nodes that are mapped to the individual positions. The second block describes the precision of the prediction of the link structure made by the role graph.

Matching of nodes to positions For each position of the role graph we extract a coordinate of the matching vector \mathbf{m} that describes how exactly the attributes of the nodes match the attributes of the position in the role graph. Thus for each position \mathbf{p}_k in the role graph the average matching quality of all nodes from the current network (V_i, E_i) that are mapped to this position is measured:

$$\mathbf{m}_k = \frac{1}{|P_k \cap V_i|} \sum_{\mathbf{v} \in P_k \cap V_i} d_W(\mathbf{v}, \mathbf{p}_k),$$

where $d_W()$ is the same weighted distance measure that was used for clustering the nodes of the ensemble.

Position sizes To measure the distribution of the nodes of the network to the different positions of the role graph we use another set of coordinates. Again we use a coordinate for each position of the role graph and encode the size of the node fraction that is mapped to a single position p_k :

$$\mathbf{m}_{r+k} = \frac{|P_k \cap V_i|}{|V_i|}$$

as a value for this coordinate. Here r is the number of nodes in the role graph.

Matching of structure prediction As described above, the second part of the matching vector is derived from the precision of the link structure prediction made in the role graph. Therefore we numerate the $\frac{1}{2}r(r+1)$ links of the role graph in the order of their appearance in the upper triangle of the (symmetric) weight matrix A : $(A_{1,1}, A_{1,2}, \dots, A_{1,r}, A_{2,2}, \dots, A_{2,r}, A_{3,3}, \dots, A_{r,r})$. Now for

each of these weights we measure the difference to the realized edges in the current network, such that for $A_{i,j}$ being the l th element in that order the corresponding coordinate for the network (V_k, E_k) becomes:

$$\mathbf{m}_{2r+l} = A_{i,j} - \frac{|\{\{a,b\} \in E_k : a \in P_i \wedge b \in P_j\}|}{\sqrt{|V_k \cap P_i||V_k \cap P_j|}}.$$

For a role graph with r nodes this yields an $\frac{1}{2}r(r+5)$ dimensional matching vector for every network in the ensemble. In the following we will use these vectors for a characterization of the ensemble.

3.3.5 Ensemble clustering As for the node attributes, the single parts of the matching vector can be weighted individually for distance measuring and therefore the clustering of the ensemble can be focused on the fit of the nodes to the positions, the number of nodes assigned to the individual positions in each network, the connection structures of the networks or any weighted combination thereof.

A problem to be conquered in this weighting is that the contribution of single parts to the distance may vary highly. For a role graph with r nodes there are $2r$ components describing node assignments and matching quality while $\frac{1}{2}r(r+1)$ components of the vector are solely describing the structure. For large values of r the description of the connection structure will necessarily dominate the distances between the resulting vectors. To solve this problem, we propose a normalization of the matching vectors such that the expected distance in the single parts is equalized. Therefore, for each of the three parts the expected distance between two networks in the ensemble is determined and its inverse is used as an additional weighting factor in the distance measure.

The result of this clustering is a partition of the ensemble, where networks in the parts are supposed to be structural similar. Now for every part we can derive individual role graphs describing only that part of the ensemble. By this means we can give a more detailed description of the ensemble and at the same time show trends that occur in the structures of the contained networks.

4 Using arbitrary attributes for node comparison

Standard algorithms for data partitioning or clustering usually need at some point at least distances between the different objects. Therefore, a necessary preprocessing step for our method is the encoding of the node attributes as vectors of real numbers. We will shortly describe different types of attributes found in such networks and methods to encode them. The intention of this encoding is to conserve distances between the possible values of the attributes before encoding. For a categorization of attribute types we follow [Stevens, 1946] and give translations of the different types defined therein into vectors.

Nominal For nominal attributes we distinguish the special case of dichotomic attributes and general ordinal attributes. Dichotomic attributes are encoded by 0 and 1, missing values by $\frac{1}{2}$. The encoding for missing values minimizes on the one hand the distance to the value that should have been in this place and assures on the other side equal distances to both existing values.

Attributes that can take values from an unordered set of n objects with pairwise equal (or unknown) distances can be naturally encoded as a vector from $\{0, 1\}^n$. For every possible value of the attribute the corresponding coordinate is set to 1 and all others remain 0. This guarantees the pairwise equal distance and allows to encode unknown values as the vector of all zeros.

Ordered For ordered values some expert knowledge is needed in order to map the different possible values into some vector space of arbitrary dimension. The problem here is that the order does not imply a fixed distance and that it may not even be possible to arrange all possible values on a one dimensional scale such that satisfactory pairwise distances are realized. This can be achieved by producing the matrix of pairwise distances of the different values with the help of some expert knowledge. Using this distance matrix and methods of nonmetric multidimensional scaling, the values can be embedded into a vector space such that the desired distances are realized. For more details on how this can be done compare e.g. [Cox and Cox, 2001].

Interval and Ratio For attributes with ordered values that conform to a linear distance the encoding is naturally given by a mapping to $[0, 1]$ or some other interval. The difference between interval and ratio types is the absence of an absolute zero value in the interval case, which is a necessity for ratio values. We handle the case of interval typed attributes as ratio typed attributes by demanding a canonical form that can be achieved by defining an absolute zero value and fixing the scale for the values of that attribute. For missing values the situation here is more complicated but can be solved by encoding them as the average over all possible values for the same reasons as in the dichotomic case.

For every node the encoded values are concatenated to a single vector containing the encoding of all attributes for this node. Consequently we have a vector that can be partitioned into blocks such that each block is used to encode an individual attribute. On this vector we can now use a distance measure that enables the weighting of the attributes by their importance as described in section 3.3.1.

5 Example analysis

5.1 General focus

We follow the approach of [Brandes et al., 2008] and classify the alters by their countries of origin combined with the country of residence, both compared with the ego situation. In the first step the alters of all networks are partitioned into four categories:

- **origin** - the alter stems from the the same country as the ego and still lives in that country
- **fellows** - the alter stems from the same country as the ego and also immigrated
- **host** - the alter lives in the country the ego immigrated to and stems from that country
- **transnationals** - all other.

We will use this partition of the nodes for the derivation of a first role graph used to describe the complete ensemble. When compared to the results of [Brandes et al., 2008] for individual networks, the outcome of this process is an averaged role graph over all networks in the ensemble.

5.2 A first result: statistic ensemble description

The first step of our analysis yields a role graph that tries to describe all networks in the ensemble at once. This first role graph is shown in figure 1. For the

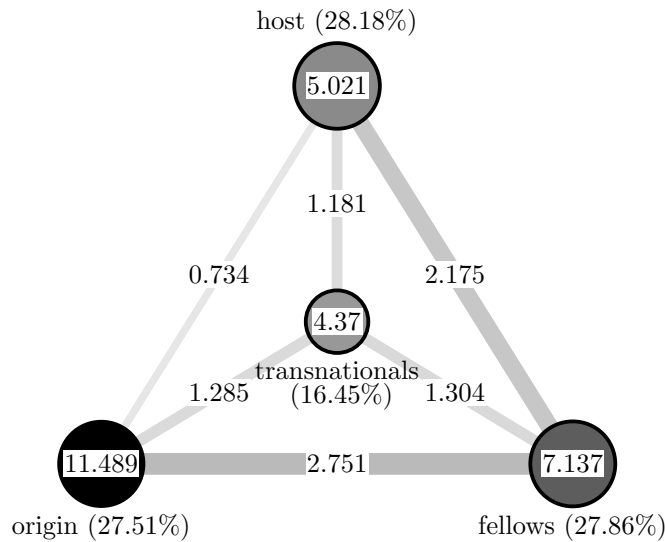


Fig. 1. Role graph describing the ensemble of personal networks. Edges and nodes are colored by density of the connections between the different groups or inner group connection density. Node sizes express sizes of the corresponding positions. The number of nodes belonging to each position is given in parenthesis.

visualization part we again rely on the former analysis and color edges by the

average degree described by them, that is the darker an edge is the higher is the average degree between the connected positions in the ensemble. For the colors of the nodes the same holds but here the average degree within a group is described. We further show the average size of the positions within the networks as the size of the position in the role graph visualization.

In general this is a very weak description of the ensemble since it is only an average and even additional measures such as standard deviation or descriptions of outliers are missing. However, a general trend can be read from this result that seems to hold throughout the ensemble. The most obvious detail is that the individual positions seem not to differ too much in size and the average degree of connections within the positions tends to exceed that between different positions.

5.3 Refining the description

As described in section 3.3.5, we can use the projection of the networks to this initial role graph to compute a matching vector that enables us to cluster the ensemble itself. However, the use of the matching vector is not limited to this single purpose. Figure 2 shows a scatter plot of the ensemble in which every network is depicted as a single point. The distances of the points in the plot approximate the distances between the matching vectors of the corresponding networks by classical scaling. For a compact explanation of classical scaling [Cox and Cox, 2001] is a good resource.

Since the initial clustering is an exact partition by group memberships, the part of the matching vector describing the matching quality of the nodes to their position contains no information and is omitted. We can though distinguish the networks by their group sizes (the fraction of nodes mapped to the positions of the role graph) and the average degree between and within the different positions. In figure 3(a) we show an analog plot based solely on group sizes and figure 3(b) is based only on the average degrees.

Clustering the ensemble The different shapes in the plot of figure 2 depict a clustering of the ensemble. Note that the plot is an optimal *approximation* of the vector distances from a space of much higher dimension than two. It is therefore plausible that even in a perfect clustering the clusters overlap when reduced to two dimensions. Analog to the plot, the clustering is based on the coordinates of the matching vector describing both, structure and sizes of the positions in the individual networks weighted equally. We use this clustering as a basis for the final step of our analysis.

Some technical details For simplicity of implementation and since a detailed examination of different clustering approaches is out of scope for this paper we chose the k-means algorithm to find an appropriate network clustering. Since we do not know in advance how many positions are optimal for the description of a given ensemble, we iterated over a range of possible cluster sizes. To take care of the non-deterministic nature of the initialization of k-means we further repeated the clustering for each size several times. We compared the clusterings resulting

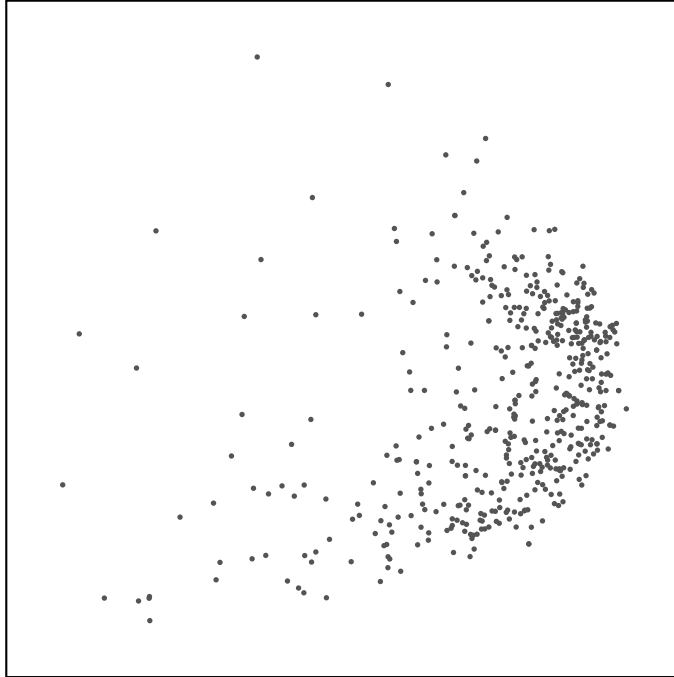


Fig. 2. Plot of the ensemble based on structure and relative position size. A clustering is depicted by the different point types.

from the different iterations and parameters with the silhouette measure to select an optimal one. We do not state that this is the optimal approach for this problem, but the intention here is to show a working example of our method. Further the method of clustering is an exchangeable part of this method and a decision of which algorithm is best suited depends on the underlying data and the purpose of the examination.

Role graphs for individual clusters For each of the clusters a new, finer role graph is determined. Therefore, all nodes in that cluster of networks have to be clustered again. Here the same method as in the first node clustering was used. Nodes were assigned to clusters matching the groups defined in 5.1 and positions for the role graphs were derived from that grouping. The role graphs for the individual groups are shown in figure 4. These finer descriptions illustrate differences in the groups of networks determined by the clustering of the previous step.

Role graph interpretations As a result of this first examination we can associate the four clusters, corresponding to the four role graphs in figure 4, with the

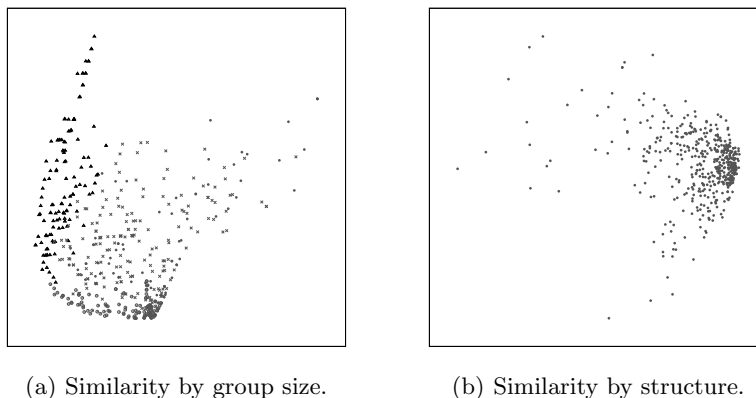


Fig. 3. Plots of the ensemble showing different aspects of similarity.

modes of acculturation proposed by [Berry, 1997]. The networks belonging to Cluster 1 (figure 4(a)) show strong separation, both with respect to nationality (most of their alters are born in the country of origin) and with respect to place of residence (most of their alters still live in the country of origin). The immigrants giving rise to the networks in the second cluster (figure 4(b)) know many people living in the host country but still show strong separation with respect to nationality since most of their contacts are classified as fellow immigrants. Cluster 3 reveals high levels of integration; the alters in these networks are well distributed over the three classes origin, fellows, and host and, in addition, these classes are well connected. Immigrants classified into cluster 4 are assimilated since they know only few alters from their country of origin but most alters stem from the host society.

6 Summary and outlook

We proposed a model that summarizes an ensemble of networks. To derive such a model from a given ensemble a method was presented that is able to focus on different aspects of the networks under examination. The presented method is not only able to derive abstract descriptions but has further applications in the exploration of network ensembles. For the concrete model derivation we focused on attributes of the nodes contained in the network. We further showed how to cluster an ensemble of networks into parts of structural similar networks. As for the node clustering, the clustering of the ensemble can be focused on different parts of the structural descriptions. Finally, by using the obtained partition a more detailed description of the ensemble can be achieved by deriving role graphs for the individual clusters. To complete our description of this method we applied it to a collection of personal networks of immigrants and showed how the obtained descriptions for the parts of the ensemble correspond to different types of acculturation.

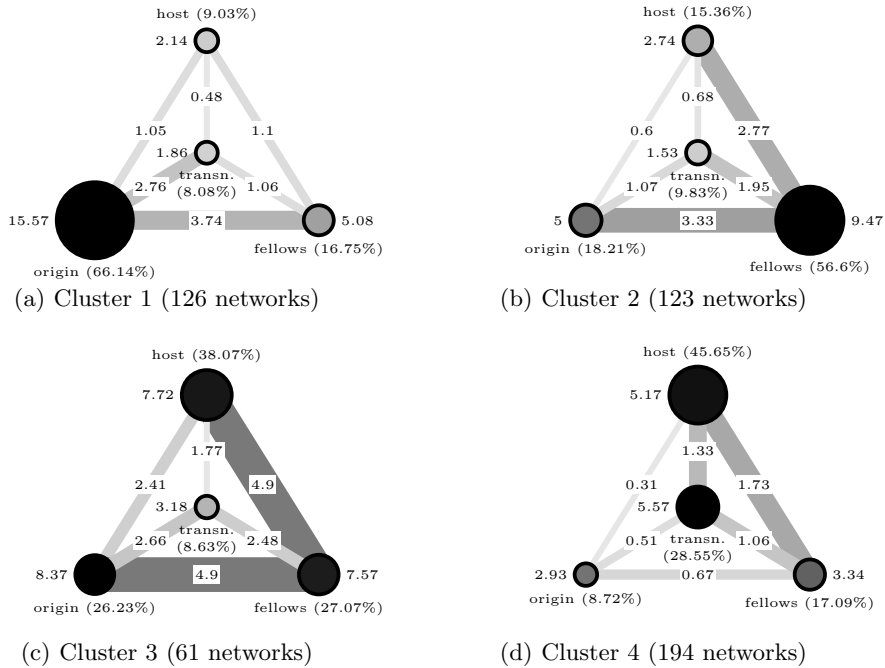


Fig. 4. Role graphs for the different clusters of the ensemble. Intra class average degrees appear outside of nodes, average degrees between classes appear on edges. The fractions of actors belonging to the positions is given in parenthesis.

Outlook The presented work could be extended on two different levels, the application level and the method itself. For the application part it would be interesting, how long the members of the different groups live in the country they immigrated to and if a relation to their situation with respect to acculturation can be shown. Also other examinations of this and other data sets are possible and would show the value of our method for explorations of networks ensembles. Especially interesting here would be the case where no previous assumptions on the outcome exist.

The extension and further development of the method itself has two different targets: provide better possibilities for the visual exploration of network ensembles and enable the method to concentrate on different aspects of the model building process. On the second point the matching of networks to role graphs and the role graph building itself is an interesting target of development. In the current version, both rely mainly on the attributes of the nodes in the networks. If we were able to shift this in the direction of connection structures, we could enable the method to extract trends on groups of nodes within the networks that are derived from structure. To extend the possibilities of exploration, an automation of the weighting process would be an interesting approach. Eventu-

ally interesting positions in the ensemble can be derived from the fact that good clusterings of the node set are possible when the clustering is focused on certain attributes. This also would extend the possibility to explore ensembles for which no assumptions on the importance of individual attributes exist.

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Notes

¹see <http://www.egoredes.net> for a description of the project and <http://www.mdlogix.com/egonet.htm> for a description of the software.