

# Static Representations of Dynamic Networks<sup>1</sup>

James Moody  
Duke University

Words (text & fn): ~6630

Figures: 6

**Abstract:** This paper presents two representations of dynamic networks that directly incorporate time into the structure of the graph. While versions of this idea have appeared as algorithmic tricks for solving dynamic network problems in the past, the substantive importance has gone largely unrecognized. By identifying a consistent way to represent dynamic networks as static graphs, we are able to appropriate all the tools and measures we have for static networks for use on dynamic networks. *Multi-slice graph* representations “stack” relational moments over time, knitting them together with “identity” arcs. *Dynamic line-graph* representations shift perspective from nodes connected by dynamic edges, to temporally-concatenated edges linked by actors. These representations do not require analysts to artificially aggregate continuous-time networks, are easy to search, visually represent, and allow us to apply well-understood network measures to temporal graphs, greatly expanding tools for analyzing dynamic social settings.

---

<sup>1</sup> This work is supported by grants NSF: HSD 0624158 and NICHD DA012831-05. The ideas presented here are the result of ongoing discussions with Dan McFarland, Skye Bender-deMoll, Martina Morris, Peter Mucha and many others who have contributed. Scott Feld deserves credit for first proposing the line-graph representation that I build on here. Thanks to jimi adams, Ryan Light and Wendy Brenildysen for comments on earlier drafts. Please direct all correspondence to James Moody at [jmoody77@soc.duke.edu](mailto:jmoody77@soc.duke.edu)

## Static Representations of Dynamic Networks

### Introduction

Despite tremendous growth in the study of social networks and numerous publications on dynamic networks, network dynamics, and network evolution (cites), there is still something vaguely problematic about capturing the full richness of temporal dynamics within relational structures. Our canonical text explicitly defines the field in terms of “lasting patterns of relations” (Wasserman and Faust 1994, p.4), and the mathematical foundations of social network research is based in graphs composed of linked sets of vertices and edges that leaves little room for temporal features such as “trajectories” or “tempo.” Paths in graph theory assume edges are present along an observed path, making it difficult to construct “structure” when edges are composed of fleeting relational events.

At its heart, this analytic unease stems from the constraints evident in network data structures, which focus on linked sets of edges and nodes. Our traditional model for dealing with dynamics of the network (distinct from dynamics *on* a network), are to label edges by time. While this is a workable solution for statistical models (particularly those that turn on Markovian assumptions), describing how the network changes or capturing trajectories in the network structure itself is hard to do. Ideally our dynamic network tools would attend *simultaneously* to the current structure of the network and the historical trajectories of nodes through social space. These tools should provide a clear ability to capture dynamic node activity that contributes to network evolution, while making it easy to capture activities that take place on the network (such as diffusion or signaling). That is, we want an approach to dynamic networks that captures the evolving structure of the social space, the action of nodes within that space, and the flow of “bits” over the changing network simultaneously.

Analysts typically focus on summary structural properties of a network, such as the degree distribution, centrality, reachability, or community structure (Wasserman and Faust 1994). Dynamic event networks complicate this work, however, since structural properties depend

crucially on how one aggregates time. Unfortunately, there are rarely good theoretical reasons for particular aggregations. When viewed over long time windows, features typically described as “structural” (paths, cycles) emerge. But, when viewed at any moment, isolated interactions appear as structure-less dyads (for a principled treatment of this problem, see Bender-deMoll and McFarland 2006). Thus while network structure is lost in a swarm of short-term isolated dyads at short time intervals, long-interval aggregations give a false sense of connectivity that neither matches diffusion potential (Moody 2002) nor reflects the unfolding social processes that create structure.

This is, of course, as much a theory problem as a measurement issue. Explicitly linking the temporal dimensions of social life with the relational dimensions would better represent reality. Just as Breiger (1974) demonstrated the duality of persons and groups through the one-mode projection of bipartite graphs; the representations described below enable analytic purchase on the social organization of enacted-time. This is an important theoretical advantage; as prior work that focused only on timing (i.e. relational duration), *or* structure (i.e. path length and clustering) is fundamentally constrained to separate network structure from social action.

Here, I describe two representations of dynamic networks that meet this goal by expanding on solutions that have previously been treated as algorithmic tricks for solving dynamic network problems (Ahuja et al 2003, Kohler et al 2002). The first stacks temporal slices of a network (white et al, Mucha et al 2010 ) by time. The key insight is that instead of labeling edges by time, we add *identity arcs* to the graph that link past nodes to their adjacent future selves in in time (Berger-Wolf and Saia 2006; Tantipathananandh et al 2007; Mucha et al 2010; Moody & Mucha 2013). The second is a line-graph projection of the first, focusing on the temporal adjacency of edges in time. In both cases, the resulting graph is easy to search in ways that respect time, standard visual representations become informative of temporal dynamics and the representation obviates the need to artificially aggregate by time. My purpose here is to point out the exceedingly general nature of this simple solution: these representations open the door to treating

*time itself* as a type of structure that intersects with relational structures in fundamental ways, opening new doors for social network science.

### **Dynamic Network Connectivity**

Our substantive questions about social networks typically rest either on diffusion or topology. In the diffusion case, we care about the network because some “bit” is transferred between nodes, with topics ranging from peer influence to disease flow across populations to synchronizing action across great distances. In these cases, a node with the bit transfers it to another (sometimes transforming it). In the topological cases, the shape of the network signals something important about the population. Questions here range over issues of social cohesion, hierarchy, or group structure to name a few. Dynamic questions turn on changes in the shape, such as the emergence of hierarchy or the consolidation of groups. In many instances these two questions are linked (how does structure affect diffusion). In all such cases, however, the fundamental building block for network measurement is a network path.<sup>2</sup>

In dynamic networks, such paths are time-ordered: a bit cannot travel over an edge that has ended, and nodes can only pass bits forward in time. Define a *time-ordered path* as a standard graph-path: a sequence of adjacent nodes and edges starting and ending with nodes, with the added stipulation that the ending time for each step along the path must be greater than or equal to the starting time of the prior step (Moody 2002). If two edges share a node and overlap in time, they are concurrent (Morris and Kretzschmar 1997). Consider figure 1, as an example.

Panel (a) provides an aggregate image for a simple dynamic network, labeling the edges with appearance order (the AB edge appears first, then after it ends, the BC edge appears and so forth. One could just as easily label by start & end times). Panel (b) provides two geodesic distance

---

<sup>2</sup> This is distinct from aspects that are purely dyadic features, such as category mixing models or degree.

matrices, the first calculated on the aggregate network structure in the standard way, the second respecting the time-order of the edges.

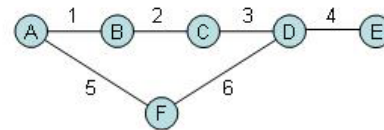
When time is ignored, the distance matrix is symmetric and complete: every pair can reach every other pair. When time is included, the geodesic path structure changes dramatically. First, many pairs are unreachable. For example, node A is 2-steps from C, but C cannot reach A, because any bit passed from C to B at time 2 cannot travel across the already-ended AB link. Thus, while all dyadic relations here are undirected, the time-ordered distance matrix is asymmetric. For connected pairs, the geodesics need not cumulate in the simple transitive manner we are used to. For example, while there is a two-step path from A to D and a 1 step path from D to E, the shortest distance from A to E is 4 steps (*not 3*) since the {DE} edge ends before the {FD} edge starts. This example highlights how ignoring time in aggregate versions of dynamic networks misrepresents fundamental features of the social structure. Any network measure that depends on path structure, such as most centrality scores and community finding algorithms, will be based on unrealizable path structures when using aggregated networks.<sup>3</sup>

Slicing the network into windows only partially helps with reachability and accuracy. Consider a version of figure 1 that cuts the network into 3 2-period time slices. This would imply mutual reachability between A and C; effectively treating the AB and BC dyads as

**Figure 1. Effect of Aggregation on Distances**

**a) Aggregate Network Structure**

*Edge labels are appearance order, each edge ends before the next one starts.*



**b) Geodesic distances**

	A	B	C	D	E	F		A	B	C	D	E	F
A	-	1	2	2	3	1	A	-	1	2	2	4	1
B	1	-	1	2	3	2	B	1	-	1	2	3	2
C	2	1	-	1	2	2	C	∞	1	-	1	2	2
D	2	2	1	-	1	1	D	∞	∞	1	-	1	1
E	3	3	2	1	-	2	E	∞	∞	∞	1	-	2
F	1	2	2	1	2	-	F	1	∞	∞	1	∞	-

*Ignoring time order*

*Time-ordered*

<sup>3</sup>This is also true for of our standard network search algorithms, including those used to find paths. We cannot use a simple breadth-first search (BFS) to find distances since paths don't cumulate in the geodesics as we'd assume. While Moody (2002) provides an (admittedly cumbersome) algorithm for searching dynamic paths, there are ways to append path and node information to BFS and depth first search (DFS) which are accurate and somewhat faster. However the static representations described below solve the algorithm problems too, since temporal reachability reduces to directed reachability.

concurrent. In general, the risk of any binning strategy is that false connectivity emerges that creates connectivity where none exists, while the risk of time-slicing is to under-estimate systemic connectivity and over-estimate within-slice connectivity.

### **Prior representation strategies for dynamic networks**

Attempts to incorporate time into structure are fairly common in network visualization, though it is rarely linked to measurement (but see de Nooy 2008). The most common way to visualize dynamic networks is to plot separate time-slices (Roy 1983), this requires much user energy to follow nodes over time in larger networks. Labeling edges with times (as above) works for small graphs, but over-emphasizes the aggregate structure leading to false impressions of connectivity.

Visualization attempts to link nodes over time by simply reserving one axis of the graph for time and the other axis for minimizing edge-crossing (McElroy et al. 2003) are somewhat difficult to read. It is difficult to know where to place nodes along the time dimension (at the start of a relation or at the end?). Having only a single dimension for representing social space is similarly constraining. A more promising approach is to following individuals through a network space over time. One can then simply plot physical or group space as a “slice” and track individuals through that space. Such representations typically use 3 dimensions to maintain the spatial layouts of the contact structure and project time in a third axis. These models use a visual cue to capture the indexing subscripts used in dynamic statistical models (Huisman and Snijders 2003; Snijders 1998). Abstracting from direct ties to group membership over time can be effective for small groups. The best recent work of this sort uses lines in the image to connect the same node over time and colorings to signify group membership (Berger-Wolf and Saia 2006; Tantipathananandh et al. 2007; Moody & Mucha 2013), which is exactly the first projection format described below.

A natural representation for dynamic networks is with dynamic visualizations, and recent work has advanced a number of tools and best-practice suggestions for animating social networks (Bender-deMoll and McFarland 2006; Leydesdorff and Schank 2008; Moody et al 2005).

Animations are particularly effective at presenting tempo and evolution of the network, and provide a good sense of action within the structure of relations. But, since viewers' visual memories are imperfect, it is difficult to track node histories over time.

### *Generalizing prior work*

We can generalize two features from prior work. First, we can build time into the network by translating temporal-order to path-order with “multi-slice networks.” This is accomplished by combining multiple relations over time (similar to familiar  $X_t$  dynamic statistical models (Butts 2008; Huisman and Snijders 2003; Snijders 1998)), but adding asymmetric “identity” arcs linking nodes to themselves over time.<sup>4</sup> Second, we can abstract from the standard nodes-as-points representation to project a *temporal line graph*, where edge events are linked to each other through nodes.

### **Defining Multi-slice representations of dynamic networks**

The simplest static representation of a dynamic network pools all relational information in high-temporal resolution. The main innovation lies in using two sorts of relations on the network. Nodes are linked to later versions of themselves with *identity arcs*. At its simplest, an identity arc is just an indicator for node-name, testifying to the connection between “John-at-noon” to “John-at-midnight.” These are arcs, not edges, since they reflect order in time: “John-at-noon” becomes “John-at-midnight.” With more complicated versions of selves, identity edges can become more complex. So, for example, if a node undergoes a fundamental change in its nature, one might

---

<sup>4</sup> I borrow the term “multi-slice network” from Mucha et al 2010. The key difference, however, is that in the multi-slice community detection routine developed there, the authors treat identity arcs as undirected edges, which is necessary for allowing a random walk to move between time slices. Here, we are seeking to use the asymmetric nature of time to order observed relations; so this is a non-trivial difference.

reasonably exclude an identity arc between “early” and “late” selves. This would be particularly likely when nodes are corporate actors – such as nations – where we distinguish between pre-revolution and post-revolution. The converse is also possible: If two identities merge into one (such as a corporate merger), then two identity arcs will point to the merged node from its earlier constituent parts and split if a node becomes multiple at later time points (such as a corporate division). The ability to easily accommodate such node transformations (in addition to population entry/exit) is a fundamental advantage over other representations. For what follows, I focus on the simpler case where nodes are unitary and identity reasonably represented as stable connections from self to self over time.<sup>5</sup>

Identities are linked to each other using whatever typical relation is collected on the population, such as friendship, “talking with”, and so forth. Thus the full graph contains three linked sets:  $G(V, E, I)$ , where  $V$  is a set of time-specific node labels  $(i_t, j_t, \dots, v_t)$ ,  $E$  are time-specific links from one set of nodes to another  $(\{i_t, j_t\})$  which will typically have both start and end times on the edge, and  $I$  is the set of (ordered) identify arcs connecting  $v_t$  to  $v_{t+1}$ . A simple example of this construction is given in figure 2 below.

In this figure, the first relational action is between B and D, starting at time 0 and ending at time 1 (two units long). B later starts a relation with A, and while it is ongoing, takes up with C. Nodes D and C engage F and E respectively after their interactions with B have ended. In the full representation (panel b), each node is represented as a separate instance in the stacked graph and thus appears at every moment they are active in the setting. Identity edges connect self at

---

<sup>5</sup> The use of the term “identity” here is in homage to H. White (1992), while I do not presume to be building the bulk of his model into this representation, this work opens a network framework that naturally allows node identity to shift.



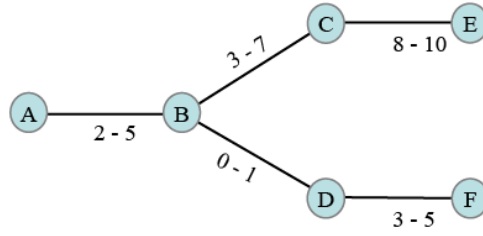
time  $t$  to self at  $t+1$ . When a node is not active, it may be removed from the graph (as with node D at time 2 in panel b).<sup>6</sup>

For most structural applications, repeating nodes and edges at each observational moment is inefficient. Given  $N$  nodes observed at  $T$  times, the full multi-slice representation will have  $TN$  nodes,  $(T-1)N$  identity arcs, and  $\sum_t(E_t)$  relational edges. We can simplify this representation considerably, without connectivity loss, by replacing nodes with no change in their tie pattern with edges, thus recording nodes and edges only when a change occurs within an edge neighborhood. At the start and end of any relation a new temporal version

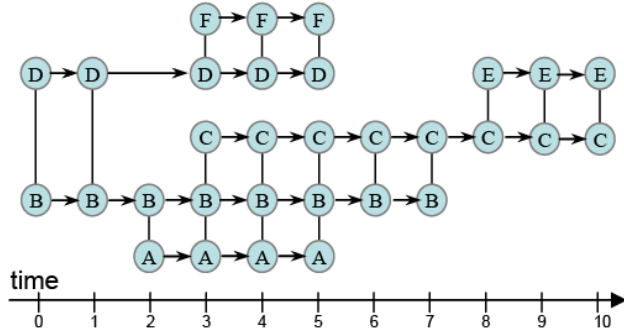
of the edge is added to the time-projected graph. If any relation changes, all edges involved with that relation are also (re)introduced. For example, in figure 2 at time 3 B initiates a relation with C, and we introduce a new BC edge. Since A is currently connected to B, the BA relation is also included at time 3. If relations are stable, they are simply linked by longer identity arcs, becoming

**Figure 2. Multi-slice representations of a dynamic network**

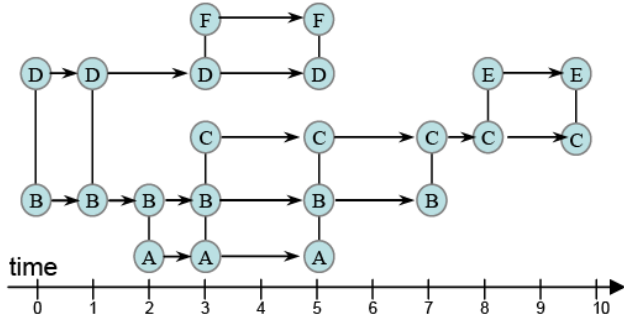
**a) Aggregate network structure**



**b) Full multi-slice Representation**



**c) Condensed multi-slice representation**



<sup>6</sup> One can distinguish active-but-isolated from inactive by including nodes who could be connected at time  $t$  but who have no relations (isolates) as a node present at  $t$  with no relational edges. Since search and reach potential are conserved through the identity edges, including or excluding isolation moments has no effect on searching in the graph.

valued by the time between relational events. I refer to this representation as a *condensed multi-slice graph*, and it is given in panel c of figure 2.

### **Advantages of the multi-slice representation**

A key advantage of this representation is that now directed paths in the graph are time-ordered paths. So here, for example, we can see that node D can reach C through a path that is two relational steps long (sum of relational edges) and takes 3 moments (sum of identity arcs)  $\{D0 \rightarrow D1 \text{---} B1 \rightarrow B2 \rightarrow B3 \text{---} C3\}$ , but there is no path from C to D. Concurrency is simple to identify as nodes with multiple edges at any time slice. Simple path-search algorithms can automatically track the number of identity vs. relational edges on a path for weighting distance. Importantly, we not only get information on geodesic distance, but distance in time. It is often useful, particularly if the bit being diffused has a fast decay rate, to know how long it takes to get from one node to another in the network.

Perhaps most importantly, the procedure generates a natural temporal expansion into structure: time is mapped onto the space of social relations, which allows us to see time in a way that might otherwise be difficult to see. To make this point clearly, consider the same aggregate contact structure with different temporal arrangements.

### *Data collected in waves*

The full-representation of the time-space graph is probably most useful when data are pre-binned due to collection (as in waves of a survey assessing current relations) or for non-event data where we expect people to have high concurrency in the relation (such as friendship). In these sorts of data, the repeated identity arcs push the entire structure into a resolvable space, and time typically emerges as a distinct dimension of the compiled network. For example, consider figure 3 below, which represents 14 waves of simulated data on a network of 32 nodes. Applying a simple 3 dimensional space-based layout to the time-space graph results in the figure in panel A, while a simple 2 dimensional layout gives us panel b.

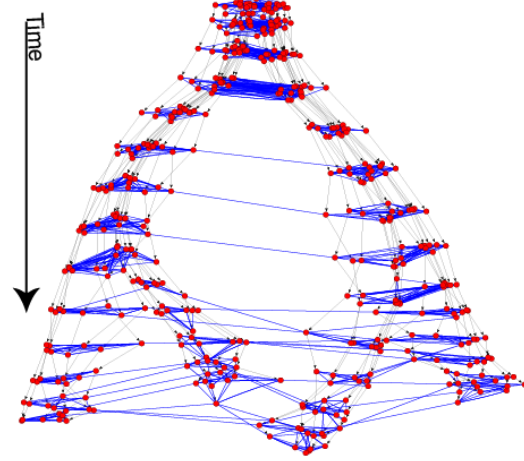
The simulation split a unified random network (time 1) into two groups (at time 5) and then split each of those groups into two again (at time 13). Within-group ties were randomly assigned, causing identity edges to cross between waves since node position changes from wave to wave in a memory-less fashion. The evolution of this 4-way split is evident in the figure, and similarly embedded in the structure of the resulting graph. Note that time is not manipulated in these figures – I have not imposed a temporal axis – it simply emerges as a function of the joint structure of relations and identity arcs.

Real data provide similar results. Consider figure 4, which applies a 3 dimensional layout for the time-space graph representation of the first 5 waves of the Newcomb fraternity data (Newcomb 1961; Nordie 1958). Since the time-dimension emerges as distinct from the relational dimensions, we can use this to our advantage to summarize how node position changes in a relational space that, because actors are anchored to their prior selves, is consistently ordered. That is, the presence of the identity edges provides a layout anchor that links the space in each wave to the prior/next wave, solving the need to impose an anchor for consistent layouts (Moody et al. 2005).

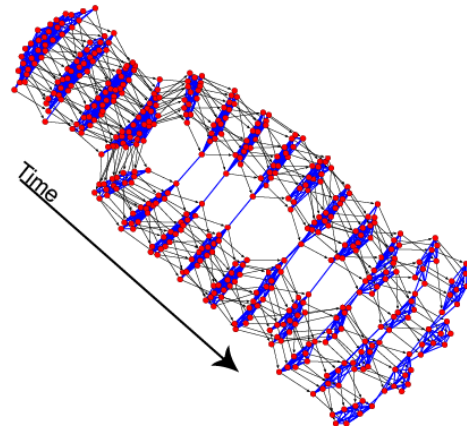
In panel b I rotate the 3d image so that time is perpendicular to the printed page. This means than “movement” in the relational dimensions of the space (xy) can be determined by following

Figure 3. TSG representations of 14 waves of data on 32 nodes

a) 3d Layout



b) 2d Layout

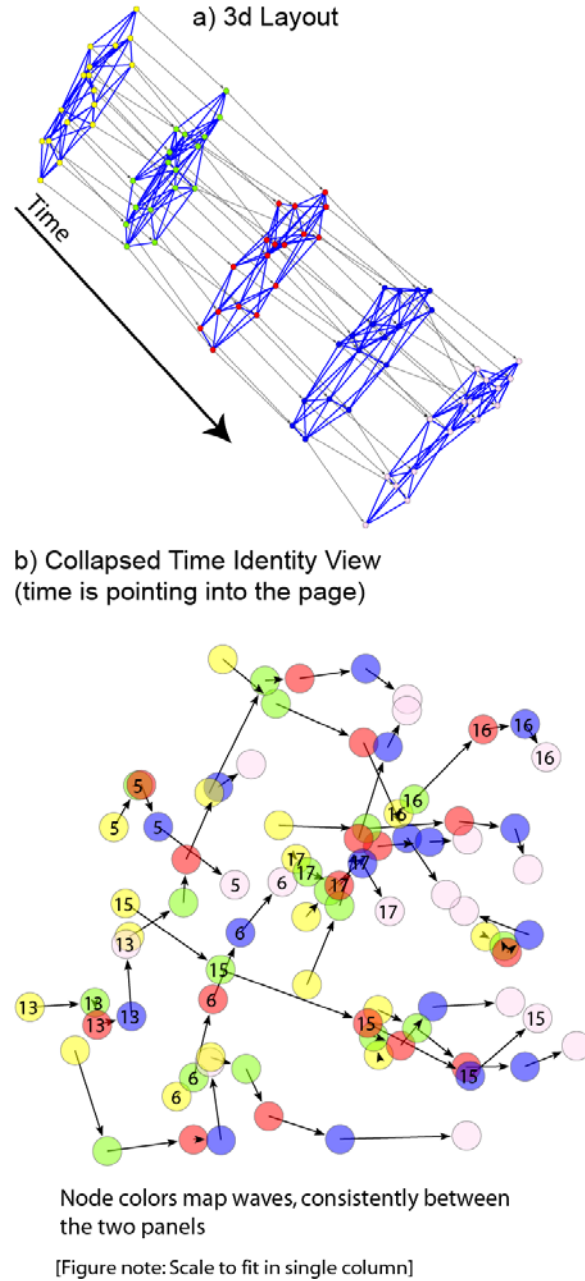


identity arcs (simpler to see if you suppress relational edges, one could also fade them substantially). By suppressing the relational edges and revealing only identity arcs, we get a sense of how people’s position in the space changes. Some nodes remain stable on either the periphery (node 13) or the center (node 17). Others migrate across the social space (node 15). Distance between pairs of nodes reflects likelihood of being jointly connected over time. For example, nodes 5 and 6 are very far apart at wave 1, but converge toward a similar point in the space by wave 6. This method should stabilize movies as well, since moving a “camera” through each wave (and interpolating position between the slices), would provide a consistent relational space for network animations.<sup>7</sup>

### *Streaming Interaction Data*

The distinctiveness of the temporal dimension in the structure of the network results from the repeated regular observations of the same nodes. When structures have a sparser time dimension, time becomes localized in the overall network and the relational space dominates. This is most likely to occur when data are

**Figure 4. TSG representations of Newcomb, first 5 waves**



<sup>7</sup> Thanks to Joshua Socolar for this suggestion.

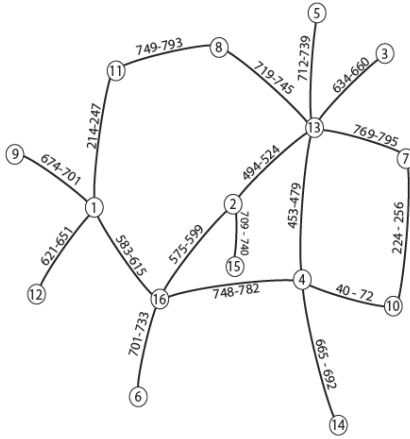
collected with continuous time-stamps on low-degree relations, such as sex ties in an STD contact network. In panel A of figure 5, for example, I present a section of a simulated sexual contact network in aggregate format with timing labeled on the edges. In panel b, we see a 2-dimensional layout for the time-space graph and panel c constrains time to a single dimension (by using start/end dates of each relations). In both time-space representations, the first and last instance of a node is labeled. Note how in panel b the topology of the graph is dominated by relational space: nodes are close to their relational partners and the “loop and pendant” structure of the aggregate graph is mirrored in the time-space graph<sup>8</sup>. This is because most nodes have only 1 or two relations and appear for only a short moment in the history of the setting.

---

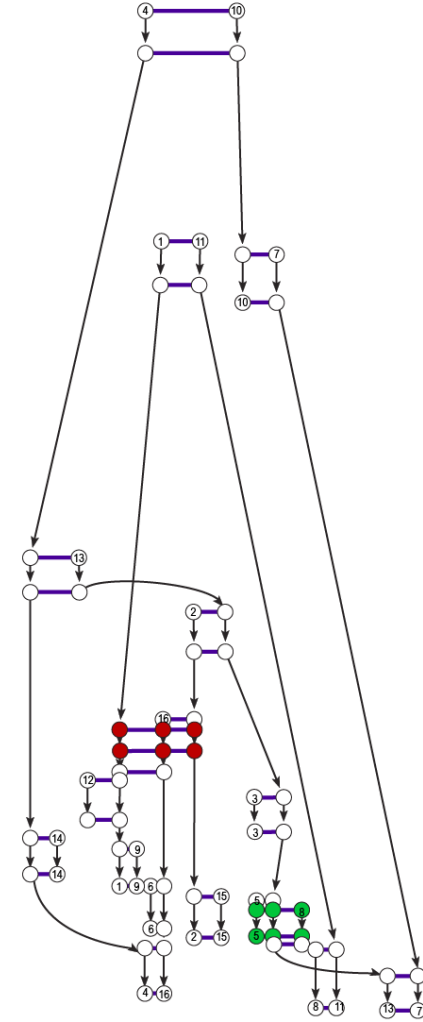
<sup>8</sup> This is also true for a 3d layout for this network.

Figure 5. TSG Example w. real-time, sparse networks

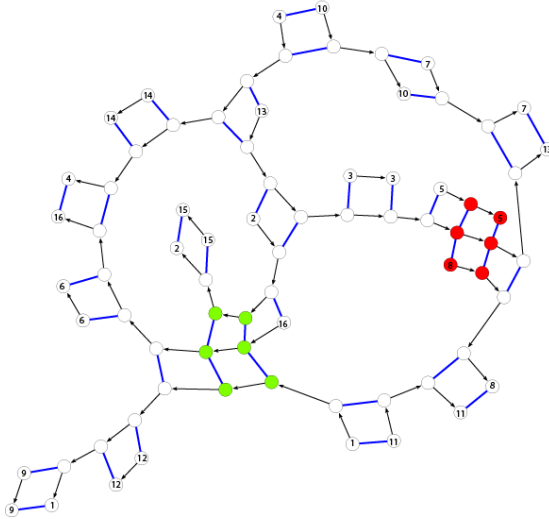
a) Agregate Contact graph with time-labeled edges



c) TSG with time-determined layout



b) TSG with space-ased layout



### Measurement on time-space graphs

It is important to emphasize that these representations are analytically useful and not just for imaging. When data are collected in real time, each strongly connected component will be composed of concurrent relations. Because of the order constraint on identity arcs, reachability in the graph directly captures diffusion potential. Simple path-search approaches (breadth-first search, depth-first search) can determine which actors could pass a bit to others. Since nodes are also labeled with time, one can easily extract not only who can reach who, but when they are reachable and for how long. When data are collected in waves (without internal time-stamps), all relations collected in the same wave must be treated as concurrent (as in the Newcomb data).

While this over-estimates reachability (as not all within-wave relations happen at the same time), it still allows one to disentangle between-wave changes in diffusion potential.

One can use the time-space networks to easily construct reachability matrices for calculating dynamic-sensitive centrality and cohesion measures. A dynamic-sensitive geodesic distance matrix can be constructed that will tell you both the number of relations on the shortest path between two nodes as well as the sum of the length of time between two nodes. These would allow us to create a time-consistent closeness centrality index or an index of “longest exposure” between pairs of nodes (Moody 2006). Finding the highest edge-betweenness scores among identity arcs would identify identity nodes *and times* that are crucial for linking others. Such measures are likely particularly important for diffusion studies, which depend on knowing how dynamics open the path-structure for movement across the population.<sup>9</sup>

### The Dynamic Line Graph

While a time-space representation intuitively retains nodes and arcs, difficulties associated with using this representation stem from having to maintain a distinction between two types of relations. Moreover, the size of the time-space graph can expand rapidly for data collected on large networks or over long time periods. We can solve some of these difficulties by creating a dynamic network projection composed of relational events linked by actors. This transformation is well known in graph theory as the *line graph* (Harary 1969). A standard line-graph  $\mathbf{L}(\mathbf{G})$  is constructed from any graph  $\mathbf{G}$ , by creating a vertex in  $\mathbf{L}(\mathbf{G})$  for every edge in  $\mathbf{G}$  and an edge in  $\mathbf{L}(\mathbf{G})$  whenever the edges in  $\mathbf{G}$  share a node.

A dynamic line graph,  $\mathbf{TL}(\mathbf{G})$ , follows the same logic, but retains the time-order of relational events. To form the dynamic line-graph, first convert all observed edges ( $\mathbf{E}_{ij}$ ) of  $\mathbf{G}$  to nodes in  $\mathbf{TL}(\mathbf{G})$ . Then add a *directed* arc between two nodes in  $\mathbf{TL}(\mathbf{G})$  whenever the corresponding edges

---

<sup>9</sup> A full examination of new dynamic centrality measures are beyond the scope of this paper and are treated in-depth in a second manuscript. Initial results suggest fairly low correlations between static and dynamic centrality measures.

in  $\mathbf{G}$  share a node. The arc will originate in the earlier edge and point towards the later edge.

This directionality is formed with a simple rule: given two edges with an overlapping node, draw a directed arc from  $\mathbf{E}_{ij}$  to  $\mathbf{E}_{jk}$  if the start time for  $\mathbf{E}_{ij}$  is earlier than the end time for  $\mathbf{E}_{jk}$ . If two edges are concurrent, this will generate reciprocal arcs in the dynamic line graph. Applying this transformation to the networks from Figures 2 and 5, generates figure 6 below (with topological rather than calendar-time layout).

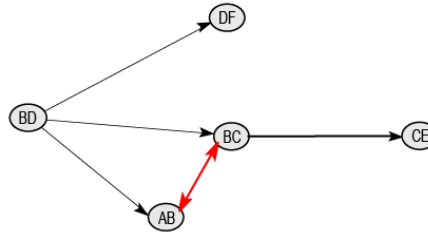
In this representation, the relational duration is collapsed into nodes (though it can easily be recorded as an attribute). We can follow an actor's trajectory through relations as chains of edges.

Concurrency in the contact graph is seen as reciprocal edges in the line graph (red double-headed arrows here).<sup>10</sup> Under a complete application of the construction rule, any edge containing node  $i$  at time  $t$  will be connected (either sending or receiving) to every other edge containing node  $i$ . We see this in panel b of figure 6, where the many closed triads capture the multiple relations of particular actors (such as  $(\{4-10\}, \{13-4\}, \{4-14\})$ ), which captures the order of 3 relations containing

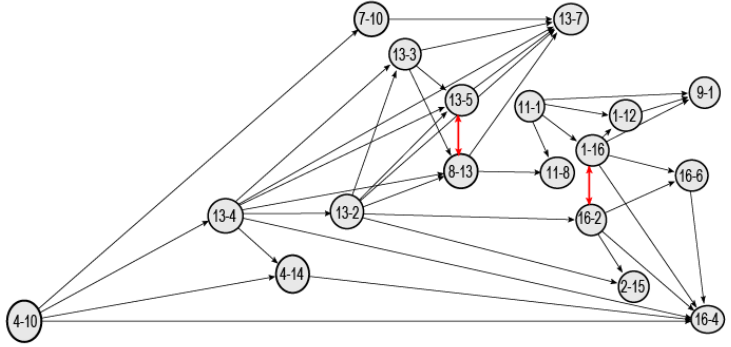
node 4). In most substantive cases, these closed triads are providing redundant information and a

Figure 6. Timed Line Graph Examples

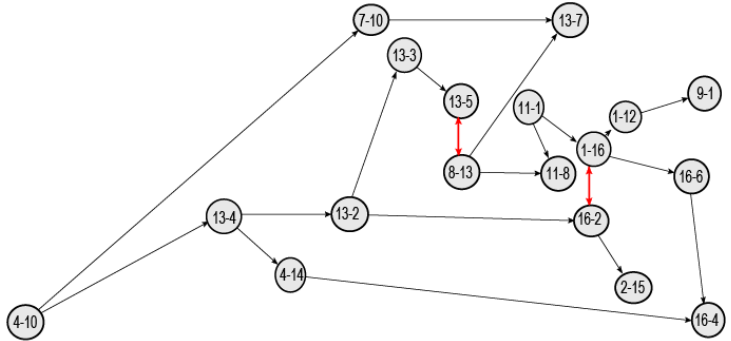
a) Timed Line Graph of Fig. 2.



b) Timed Line Graph of Fig. 5



c) Recency Reduced Timed Line Graph of Fig 5



<sup>10</sup>One could value the edges in a reciprocated arc to reflect duration of the overlap from each sender's point of view. There are many types of concurrencies that can have different implications for diffusion potential (Morris et al 2008). These could likely be captured as different weights in the line-graph.



“recency” reduction will be more efficient. After constructing the line graph as above, remove all but the most recent non-reciprocated unique incoming edges to every node.<sup>11</sup> A node will thus retain all reciprocal ties plus arcs to the most recent relational events of each composite actor. A node will have two incoming arcs only if both actors that comprise that node were active earlier. A node will have no incoming arcs if both actors appear in the population at this time. This reduction is presented in the third panel of figure 6.

Substantively, the recency reduction means that since each arc in the line-graph is an actor in the contact graph, any “bit” an actor has to pass is carried by the person and thus “through” all of the prior relations along the path constructed by that actor. This is a reasonable model for bits that are reasonably contained in actors (like STDs or ideas). But, one can imagine contexts where this assumption is less useful. If, for example, a bit of information is only *activated* in one relational context, then we may want to link to the most recent compatible relation. So, a bit of information passed from an ex-romantic partner may not be appropriately passed to your current partner. Or, the relational context may dictate a notification order (Ryan 2006) based on actor roles that could be signaled by the joint membership of an edge within the same class (“family” vs. “work” relations, with overlaps represented as multiple instances of the same person). One strength of the line-graph representation is that by highlighting connections among relations, we can more easily theorize mechanisms driving transmission across relational types. Actors become differentiated across time, and may behave differently at each moment.

Reflecting the underlying duality of events and actors, the recency reduction is exactly the same as replacing every relation-specific 4-node “box” in the condensed time-space graph (the starting tie and two nodes, the two identity arcs linking actors to their later selves, and the two ending nodes and tie; such as the first relation between 4 and 10 at the top of figure 5c) with a single node, retaining only the identity edges between the boxes. Thus, comparing the topology

---

<sup>11</sup> Doing the reduction *after* the full projection is necessary to retain reciprocated edges, which are needed for proper time-ordered path search.

of panel c in figure 6 to panel c of the time-space graph in figure 5, we find the exact same structure. But, the line-graph representation uses 39 data elements (sum of edges and nodes) while the condensed time-space representation uses 216. This is a significant reduction in complexity that retains all of the underlying dynamic contact topology. This reduction in complexity, combined with the strongly time-ordered nature of the resulting graph, makes it simpler to visualize large dynamic networks with the dynamic line graph than with the time-space representation.

#### *Measurements on the dynamic line graph.*

The switch from nodes-as-actors to nodes-as-relations requires a change in measurement perspective. What can we learn from this new representation? First, reachability and diffusion can be directly mapped with simple path-search algorithms. If there is a path from one node in the line graph to another, then the constituent actors in the contact graph can reach each other in time. This alleviates the need for complex search algorithms to calculate reachability in dynamic networks and makes it much simpler to build time-sensitive measures.

More generally, the line-graph transformation means that indices constructed on the nodes of the line-graph refer to *edge-moments* in the contact graph. As such, we can easily identify central *events* in the dynamic network. For example, a node with high centrality in the line-graph is an event in the actor-network that uniquely links together other events. On the other hand, edge metrics on the line-graph capture time-specific actor contributions to the event-structure. For example, high edge-betweenness would identify actor-moments that bridge distinct groups. Applying node partitioning algorithms to the line graph will partition events in the contact graph, allowing actors to play different roles in contact structures over time. Any topological index (path length, clustering, connectivity, etc.) on the graph will now capture differences in the intersection of relational events through actors over time. Thus two graphs with the exact same dyad structure but different relation timings will have very different dynamic line-graph

topologies. While the move to a line-graphic representation is a fairly dramatic shift from our everyday understanding of networks. The two key advantages of the dynamic line graph are first that it reduces graph complexity while maintaining both contact and temporal structure. Second, the resulting graph fits theoretical approaches that take relations as primary and actors as incident (Callon et al 1986; Emirbayer 1997; Mayhew 1980; 1981).

## **Implications**

A consistent challenge for analyzing dynamic networks is how to balance contact action, where events may occur sporadically over long reaches of time, against pairing structure that aggregates into something we recognize as a network (Butts 2008; Huisman and Snijders 2003; Moody et al. 2005; Snijders 1998). The most common solutions are to create aggregate temporal slices, animate the network, or build algorithms and statistical models resting on edge time labels. Time binning is often arbitrary leading to misleading comparisons of aggregate slices that provide minimal information on the history and trajectory of networks. Animations excel at showing the active tempo of change in a network, but are similarly plagued by local time binning, lapses in viewer memory, and provide minimal substrates for measurement. Statistical approaches often work with limited dynamic information, modeling either changes in summary values of the panel or specific tie patterns as a function of prior waves, making it difficult to capture nuances of time-event intersection.

The two representations presented here overcome these difficulties. Each representation can accommodate instantaneous relational events, thus avoiding time aggregation questions. Social contact is retained as path order in the dynamic line graph, and repeated edge-instances in the space-time graph. By building both time and contact into the same structure, we gain the ability to analyze their intersection, decompose event-structures by time (activity tempo, contact rates, historical bursts) and actors (relational-trajectories, movement through social space over time) and characterize entire structures by the topology of contact-events.

### *Network dynamics: durable relations or relational events?*

Both representations include a notion of relational *duration*: relations are edges between nodes with (potentially multiple) start and end times. The duration assumption is that a relation is “on” for the duration of the edge. While this is another form of temporal aggregation, since most social processes are episodic, it is more a convenience than a necessity. One could construct either network representation in instantaneous time, rather than with durations. Doing so, however, typically replaces network concurrency with cycles (at least for networks composed of interaction events, such as speaking, writing, having sex, etc.).<sup>12</sup> As such, it oftentimes makes sense to employ some common sense durational aggregation. If a man has a wife and a mistress, we want to capture these as *overlapping relations*, not separated events. In practice, the dynamic line graph and the time-space graph both handle repeated relations; so long gaps between relational events on the same pair can be substantively treated as distinct relations.

Practically, if you have relations that are substantively continuous (such as love, esteem, or friendship) rather than relational events (such as sexual contact, doing favors, or talking-with), then we will typically have dense connected networks at each moment (high concurrency), and the time-space representation will be most appropriate. The dimensional-stability imposed by identity arcs builds an orienting context for the social space, simplifying the process of linking micro-position to macro-structural change. If, instead, you have episodic relations that occur in fine-grained time, such as sexual contacts or email exchanges, the dynamic line graph representation will likely be more informative and efficient.

### *Relational time or clock time?*

---

<sup>12</sup> Repeated relations in an instantaneous version of a network will create cycles in the time-ordered graphs of the form A – B – C – B – A – B – C .... For the duration of the overlapping relations between AB and BC (Moody forthcoming). Any durational definition that avoids creating excess concurrency is acceptable, and simple rules such as asking when a respondent last had contact with an alter will typically work.

Traditional models for dynamic networks must force a single time metric on relations (sex partners in the last 6 months, all friends on the day of the interview) or a fixed moving window for animations over the life of the network. But actors may *experience* the network at different tempos. So, while a 6-month window is likely too long for meaningful structural effects in the behavior of a commercial sex worker, it is too short for a middle-aged pastor. We can alleviate this problem, at least somewhat, by asking about event sets (i.e. last 3 partners), but this contorts temporal and network structure by constraining degree.

These representations make a single temporal aggregation unnecessary. Precisely because the line-graph dispenses with relational duration, we can retain order without equating time. A local event network metric (a version of transitivity or ego-network density, say), will thus encompass different time-horizons for each actor, as it takes differing times for local relations to cumulate around a particular nodes. This should allow us to build models for network activity based on local event structure that has fundamentally different tempos across the population. For theoretical models that work on local perceptions of temporal events (such as sequences of violence (Balian and Bearman 2008)), this may be a more useful analytic frame than those representations which focus on universal time-scales.

### *Positions in dynamic social space*

The main difference between the time-space graph and the dynamic line-graph is the meaning of a node. In the first representation, typical notions of nodes-as-actors are retained and identity is a consistent thread linking nodes to their prior selves. This is a model that matches person-centered theoretical approaches and is thus likely to resonate with many social scientists. The line-graph inverts our understanding of nodes and edges, linking relational events to each other and disaggregating actors across time. Of course, that the reduced form versions of these two representations converge in basic topology is an important reminder of the fundamental duality of time and relations in dynamic networks. Both representations capture the fundamental features of

events comprised by chains of actors linked over time. These representations do, however, highlight interesting questions about how network models capture social space and the position of a self in that space over time.

Communities, for example, are often conceptualized as stable collections of actors who reside in a region of social space identified by dense connections to each other. These two representations make it possible to conceive of communities as time-specific aggregations of repeated interaction events. That is, one could imagine using a community-finding algorithm on a large time-space graph and finding very different time-bounds for communities of actors. In areas like the sociology of science, this could be used to much advantage for comparing specialties with different time-to-discovery horizons. The key insight, of course, is that communities are bounded by sparse regions of relational events; but this bound is both temporal and spatial. This is particularly intriguing in the line-graph version, where a community clustering algorithm could easily spread people across many different substantive event-spaces.

As with the Newcomb example above, these models make it fairly simple to identify micro movements through an evolving macro-level social space. We can watch subgroups split, people move across otherwise stable groups and so forth directly. We should similarly be able to build structural models of these graphs with tools designed for measuring structures. Exponential Random Graph Models (ERGMs) for static networks are now developing rapidly (Handcock et al 2003; Snijders, et al 2005), and this representation will let us borrow from that tool directly. One need simply construct micro-models of the local event structure to apply the tool (which, admittedly may not be trivial). In principle, this would enable models of self and identity to link personal history to community network structure.

## **Summary and conclusion**

This paper has presented two ways to directly incorporate time and relations within the same structure. The time-space graphs link actors to themselves through identity arcs, creating a

strongly time-ordered graph. The dynamic line graph projects connections between relations through the actors involved in both. In both cases, we have a single graph object that captures both time and relation, and this object is easily searched with common flow algorithms making it possible to extend all of our network measures from likely-misleading aggregate constructions to time-appropriate measures. Because the result is ultimately “just” a graph, all of our measures and modeling tools that apply to static networks can be applied (albeit with new meaning!), to dynamic graphs. In settings with high concurrency, such as waves of ongoing affective relations, the time-space representation may be the simplest way to capture relations and time simultaneously. In settings with streaming interactions occurring sporadically in real time, the line-graph representation may be most appropriate, as it negates any need for relational aggregation.

The representations suggested here also open the door for a host of new measurement applications. Ranging from typical flow problems associated with diffusion to role / position models. So, for example, preliminary work on new time-based centrality measures suggests that calculating centrality on aggregated versions of dynamic networks leads to significant bias (Moody 2006). One can imagine that measures for clustering, hierarchy, blocks or cohesion, which all similarly rest on an underlying path structure, will be similarly biased. While there is clearly a good deal of intuition left to build with these representations, they provide a simple starting point to carry our hard-earned structural intuitions from static network modeling directly into dynamic network models.

## Reference List

- Ahuja, Orlin, Pallottino & Scutella. 2002. "Dynamic shortest paths minimizing travel times and costs." *Networks* 41:197-205, 2003.
- Balian, H. and P. S. Bearman. 2008. "Chains of Violence: Continuity and Change in Northern Ireland." *Manuscript*, Columbia University.
- Bender-deMoll, S. and D. A. McFarland. 2006. "The Art and Science of Dynamic Network Visualization." *Journal of Social Structure* 7(2).
- Berger-Wolf, T. and J. Saia. 2006. "A Framework for Analysis of Dynamic Social Networks." *Proc. KDD '06* :523-28.
- Breiger, R., K. Carley, and P. Pattison. 2003. *Dynamic Social Network Modeling and Analysis*. Washington DC: National Research Council of the National Academies.
- Breiger, R. L. 1974. "The Duality of Persons and Groups." *Social Forces* 53:181-90.
- Burk, W. J., C. E. G. Steglich, and T. A. B. Snijders. 2007. "Beyond Dyadic Interdependence: Actor-Oriented Models for Co-Evolving Social Networks and Individual Behaviors." *International Journal of Behavioral Development* 31:397-404.
- Butts, C. T. 2008. "A Relational Event Framework for Social Action." *Sociological Methodology* :Forthcoming.
- Callon, M., J. Law, and A. Rip. 1986. *Mapping the Dynamics of Science and Technology*. Sheridan House.
- Centola, D., V. M. Eguiluz, and M. W. Macy. 2007. "Cascade Dynamics of Complex Propagation." *Physical A* 374:449-56.
- de Nooy, Wouter. 2008. "Signs Over Time: Statistical and Visual Analysis of a Longitudinal Signed Network." *Journal of Social Structure* 9(1).
- Doreian, P. 1980. "On the Evolution of Group and Network Structure." *Social Networks* 2:235-32.
- . 1986. "On the Evolution of Group and Network Structure II: Structures Within Structures." *Social Networks* 8:22-64.
- Doreian, P. and F. Stockman. 1996. *Evolution of Social Networks*. New York: Gordon and Breach.
- Emirbayer, M. 1997. "Manifesto for Relational Sociology." *American Journal of Sociology* 103:281-317.
- Erten, Harding, Kobourov, Wampler & Yee. 2004. "Exploring the computing literature using temporal graph visualization." *Proc. 16th Conf. Vis. Data Anal. (VDA '04)*, SPIE vol. 5295:45-56
- Epstein, J. and N. Karweit. 1983. *Friends in School: Patterns of Selection And Influence in*



*Secondary Schools*. Academic Press.

Feld, S. L. 1997. "Structural Embeddedness and Stability of Interpersonal Relations." *Social Networks* 19(1):91-95.

Friedkin, N. E. 1998. *A Structural Theory of Social Influence*. Cambridge: Cambridge.

Hallinan, M. T. 1978. "The Process of Friendship Formation." *Social Networks* 1:193-210.

Hallinan, M. T. and W. N. Kubitschek. 1990. "The Formation of Intransitive Friendships." *Social Forces* 69(2):505-19.

Handcock, Mark S., David R. Hunter, Carter T. Butts, Steven M. Goodreau, and Martina Morris. 2003. *Statnet*: Software tools for the Statistical Modeling of Network Data. URL <http://statnetproject.org>.

Harary, F. 1969. *Graph Theory*. Reading, Massachusetts: Addison-Wesley.

Huisman, M. and T. A. B. Snijders. 2003. "Statistical Analysis of Longitudinal Network Data With Changing Composition." *Sociological Methods and Research* 32:253-87.

Hummon, N. P. and P. Doreian. 2003. "Some Dynamics of Social Balance Processes: Bringing Heider Back Into Balance Theory." *Social Networks* 25:17-49.

Köhler, Langkau & Skutella. 2002. "Time-expanded graphs for flow-dependent transit times." *Proc. 10th Europ. Symp. Alg. (ESA '02)*, Springer LNCS 2461:599-611.

Leenders, R. Th. A. J. 1995. "Models for Network Dynamics: A Markovian Framework." *Journal of Mathematical Sociology* 20:1-21.

Leydesdorff, L. and T. Schank. 2008. "Dynamic Animations of Journal Maps: Indicators of Structural Change and Interdisciplinary Developments." *Journal of the American Society for Information Science and Technology* 59:1810-1818.

Mayhew, B. H. 1980. "Structuralism Versus Individualism: Part I, Shadowboxing in The Dark." *Social Forces* 59:335-75.

———. 1981. "Structuralism Versus Individualism: Part II, Ideological and Other Obfuscations." *Social Forces* 59:627-48.

McElroy, P. D., R. B. Rothenberg, R. Varghese, R. Woodruff, G. Minns, L. A. Labert, S. Muth, and R. Ridzon. 2003. "A Network-Informed Approach to Investigating a Tuberculosis Outbreak: Implications for Enhancing Contact Investigations." *International Journal of Tuberculosis and Lung Disease* 7:S486-S493.

Moody, J. 2002. "The Importance of Relationship Timing for Diffusion." *Social Forces* 81:25-56.

———. 2006. "Watching Time Fly: Diffusion & Visualization on Dynamic Networks." *American Sociological Association* (Toronto, ASA).

Moody, J. forthcoming. "Evolution and Diffusion Over Dynamic Networks." *Oxford Handbook of Analytic Sociology*, Peter Hedstrom and Peter S. Bearman. Oxford: Oxford University

Press.

- Moody, J., D. A. McFarland, and S. Bender-deMoll. 2005. "Dynamic Network Visualization." *American Journal of Sociology* 110:1206-41.
- Morris, M. and M. Kretzschmar. 1995. "Concurrent Partnerships and Transmission Dynamics in Networks." *Social Networks* 17(Special Issue. Social Networks and Infectious Disease: HIV/AIDS):299-318.
- . 1997. "Concurrent Partnerships and the Spread of HIV." *AIDS* 11:641-48.
- Nakao, K. and A. K. Romney. 1993. "Longitudinal Approach to Subgroup Formation: A Re-Analysis of Newcomb's Fraternity Data." *Social Networks* 15:109-31.
- Newcomb, T. M. 1961. *The Acquaintance Process*. New York: Holt, Rinehart and Winston.
- Nordie, P. 1958. "A Longitudinal Study of Interpersonal Attraction in a Natural Group Setting." Unpublished Dissertation, University of Michigan, Department of Psychology.
- Powell, W. W., D. R. White, K. W. Koput, and J. Own-Smith. 2005. "Network Dynamics and Field Evolution: The Growth of Interorganizational Collaboration in the Life Sciences." *American Journal of Sociology* 110:1132-205.
- Roy, W. G. 1983. "The Unfolding of the Interlocking Directorate Structure of the United States ." *American Sociological Review* (248-257).
- Ryan, D. 2006. "Getting the Word Out :Notes on the Social Organization of Notification." *Sociological Theory* 24:228-54.
- Snijders, T. A. B. 1998. "Models for Longitudinal Social Network Data." manuscript University of Groningen.
- Snijders, T. A. B. 2001. "The Statistical Evaluation of Social Network Dynamics." *Sociological Methodology* :361-95.
- Snijders, T. A. B., P. E. Pattison, G. L. Robins, and M. S. Handcock. 2005. "New Specifications for Exponential Random Graph Models." *Sociological Methodology* 35.
- Steglich, C. E. G., T. A. B. Snijders, and M. Pearson. 2007. "Dynamic Networks and Behavior: Separating Selection From Influence." *Manuscript*.
- Suitor, J. J., B. Wellman, and D. L. Morgan. 1997. "It's About Time: How, Why and When Networks Change." *Social Networks* 19(1):1-7.
- Tantipathananandh, C., T. Berger-Wolf, and D. Kempe. 2007. "A Framework for Community Identification in Dynamic Social Networks." *Proc. KDD '07* :717-26.
- Wasserman, S. and K. Faust. 1994. *Social Network Analysis*. Cambridge: Cambridge University Press.
- Watts, D. J. and S. H. Strogatz. 1998. "Collective Dynamics of 'Small-World' Networks." *Nature* 393:440-442.

White, H. C. 1992. *Identity and Control. A Structural Theory of Social Action*. Princeton, New Jersey: Princeton University Press.